Day 83 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 3 kg is pulled along rough horizontal ground by a horizontal force of 8.85 N. It accelerates at 0.5 m s^{-2}. Take g = 9.8 m s^{-2}.
Find the coefficient of friction between the crate and the ground.

R = mg = 3 × 9.8 = 29.4
Resultant: 8.85 − F = 3 × 0.5, so F = 7.35
F = μR: μ = 7.35 ÷ 29.4
Answer: 0.25

2 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 - 8x + 12 \geq 0$.

Factorise: $(x - 6)(x - 2) \geq 0$
Critical values: x = 2 and x = 6
Sketch the ∪-shaped parabola: $x \leq 2$ or $x \geq 6$
Answer: $x \leq 2$ or $x \geq 6$

3 Interpreting regression (AQA, Edexcel, OCR, MEI, Eduqas)

Regression line (x from 2 to 18): y = 60 − 2.5x.
x is the temperature, y the gas used per day.
(a) Interpret the gradient.
(b) Estimate y when x = 21 and comment.

(a) The gradient: for each extra °C rise in temperature, y is predicted to decrease by 2.5
(b) y = 60 + (−2.5) × 21 = 7.5
x = 21 lies outside the range of the data (extrapolation), so the estimate may be unreliable
Answer: (a) y decreases by 2.5 per extra °C rise in temperature (b) 7.5, unreliable (extrapolation)

4 Quotient rule (All boards)

Given that $y = \frac{\ln x}{x^2}$,

find $\frac{dy}{dx}$, and its value when x = 1.

Quotient rule with u = ln x, $v = x^2$
$u' = \frac{1}{x}$, $v' = 2x$
$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{1 - 2\ln x}{x^3}$
When x = 1: $\frac{dy}{dx} = 1$
Answer: $\frac{dy}{dx} = \frac{1 - 2\ln x}{x^3}$; value 1

5 Conditional probability formula (All boards)

P(B) = 0.4, P(A | B) = 0.6 and P(A) = 0.6.
(a) Find P(A ∩ B).
(b) Find P(B | A).
(c) Find P(A ∪ B).

(a) P(A ∩ B) = P(A | B) × P(B) = 0.6 × 0.4 = 0.24
(b) $P(B \mid A) = \frac{0.24}{0.6} = 0.4$
(c) P(A ∪ B) = 0.6 + 0.4 − 0.24 = 0.76
Answer: (a) 0.24 (b) 0.4 (c) 0.76

6 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 3 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$.

The coefficient of friction is 0.2. Take $g = 9.8$ m s^{-2}. Find the acceleration down the plane, and the distance moved in 3 s.

$\cos\alpha = 12/13$, $\sin\alpha = 5/13$

$R = mg\cos\alpha$, $F = \mu R$

$ma = mg\sin\alpha - \mu mg\cos\alpha$, so $a = 9.8(5/13 - 0.2 \times 12/13)$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 1.96 \times 3^2$

Answer: 1.96 m s^{-2}; 8.82 m

7 Equations using $\sin^2x + \cos^2x = 1$ (All boards)

Solve $5\cos^2x - 3\sin x - 5 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2x = 1 - \sin^2x$: $-5\sin^2x - 3\sin x = 0$, i.e. $5\sin^2x + 3\sin x = 0$

$(5\sin x + 3)(\sin x) = 0$, so $\sin x = -\frac{3}{5}$ or 0

$\sin x = -\frac{3}{5}$: $x = 216.9°, 323.1°$

$\sin x = 0$: $x = 0.0°, 180.0°, 360.0°$

Answer: x = 0.0°, 180.0°, 216.9°, 323.1°, 360.0°

8 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(7, −4) and B(−5, 2), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:

$49 + (h + 4)^2 = 25 + (h - 2)^2$

Expanding and cancelling h^2: $8h + 65 = -4h + 29$

$h = -3$

$r^2 = CA^2 = 50$

Answer: centre (0, −3), $x^2 + (y + 3)^2 = 50$

9 Rates of change (connected rates) (All boards)

The area of a square increases at a constant rate of 6 cm^{2}s^{-1}.
Find the rate of increase of the side length when the side is 5 cm.

$A = x^2$, so $\frac{dA}{dt} = 2x\frac{dx}{dt}$

$6 = 2 \times 5 \times \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{3}{5}$

Answer: $\frac{3}{5}$ cm/s

10 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x - 4)(x + 3)(x - 3) < 0$.

Critical values: $x = -3$, $x = 3$, $x = 4$

The cubic has a positive x^3 term, so the graph goes from bottom left to top right
and crosses the axis at each critical value.
Reading from the sketch: $x < -3$ or $3 < x < 4$

Answer: $x < -3$ or $3 < x < 4$