Day 82 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Inequalities with exponents and logs (All boards)

Solve $4^x > 174$, giving your answer to 3 s.f.

Take logs: $x \ln 4 > \ln 174$

$x > \frac{\ln 174}{\ln 4} = 3.721$

Answer: $x > 3.72$

2 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3x - 1 \geq x - 6$.

$2x \geq -5$

Divide by 2: $x \geq -\frac{5}{2}$

Answer: $x \geq -\frac{5}{2}$

3 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int e^{5x}\,dx$,

(b) the exact value of $\int_0^1 e^{5x}\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{e^{5x}}{5} + c$

(b) $\left[\frac{e^{5x}}{5}\right]_0^1$

$= \frac{e^5}{5} - \frac{1}{5}$

Answer: (a) $\frac{e^{5x}}{5} + c$ (b) $\frac{e^5}{5} - \frac{1}{5}$

4 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A golf ball is hit from the ground at 49 m s^{-1} at angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$.

Take g = 9.8 m s^{-2}.

Find (a) the greatest height, (b) the speed of the ball at the greatest height.

Vertical component 29.4, horizontal component 39.2

(a) $0 = 29.4^2 - 2 \times 9.8 \times H$, $H = 864.36 \div 19.6$

(b) At the top the vertical velocity is zero, so the speed is the horizontal component.

Answer: (a) 44.1 m (b) 39.2 m s^{-1}

5 Grouped data: mean and median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises 46 measurements of d.

(a) Estimate the mean.

(b) Estimate the median using linear interpolation.

(a) Using mid-points: $\Sigma fx = 960$, so mean = $960 \div 46 = 20.9$

(b) $\frac{n}{2} = 23$. Cumulative frequency before the median class = 13

Median class 10 to 20. Median = $10 + \frac{10}{11} \times 10 = 19.1$

Answer: mean $\approx$ 20.9, median $\approx$ 19.1

6 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of

$(1 - x)(1 + x)^5$.

$(1 + x)^5$ has x^1 coefficient 5 and x^2 coefficient 10

x^2 term of the product = $1 \times 10 + (-1) \times 5$

$= 5$

Answer: 5

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7 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 10, frequency 12, is a bar 2.5 cm wide, 2.4 cm tall.
(a) Find the bar height for width 40, frequency 15.
(b) Find the frequency of a 3 cm by 2 cm bar.

Area of bar $\propto$ frequency: $2.5 \times 2.4 = 6$ cm^2 represents 12
so 1 unit of frequency = 0.5 cm^2
(a) Width = 10 cm, area = $0.5 \times 15 = 7.5$ cm^2, height = $7.5 \div 10 = 0.75$ cm
(b) Area = $3 \times 2 = 6$ cm^2, frequency = $6 \div 0.5 = 12$
Answer: (a) 0.75 cm (b) 12

8 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 5 kg is pulled across smooth horizontal ice by a rope with tension 40 N, inclined at α above the horizontal, where $\tan\alpha = \frac{4}{3}$.

Take g = 9.8 m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos\alpha = 3/5$, $\sin\alpha = 4/5$
(a) Horizontal: $40 \times 3/5 = 5a$
(b) Vertical: $R + 40 \times 4/5 = 5 \times 9.8$
Answer: (a) 4.8 m s^{-2} (b) 17 N

9 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{4x + 15}{(x + 3)(x + 4)}$ in partial fractions.

(b) Hence evaluate $\int_0^1 \frac{4x + 15}{(x + 3)(x + 4)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{4x + 15}{(x + 3)(x + 4)} \equiv \frac{A}{x + 3} + \frac{B}{x + 4}$, so $4x + 15 \equiv A(x + 4) + B(x + 3)$
$x = -3$: $A = 3$; $x = -4$: $B = 1$
(b) $\int_0^1 \left(\frac{3}{x + 3} + \frac{1}{x + 4}\right) dx = [3\ln(x + 3) + \ln(x + 4)]_0^1$
$= 3\ln\frac{4}{3} + \ln\frac{5}{4} = \ln\left(\frac{80}{27}\right)$

Answer: (a) A = 3, B = 1 (b) $\ln\left(\frac{80}{27}\right)$

10 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{5x^2 + 22x + 36}{(x - 3)(x + 4)^2}$ in partial fractions.

$\frac{5x^2 + 22x + 36}{(x - 3)(x + 4)^2} \equiv \frac{A}{(x - 3)} + \frac{B}{(x + 4)} + \frac{C}{(x + 4)^2}$
$5x^2 + 22x + 36 \equiv A(x + 4)^2 + B(x - 3)(x + 4) + C(x - 3)$
$x = 3$ gives $A = 3$; $x = -4$ gives $C = -4$
Coefficients of x^2: $5 = A + B$, so $B = 2$
Answer: $\frac{3}{(x - 3)} + \frac{2}{(x + 4)} - \frac{4}{(x + 4)^2}$