Day 81 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lorry of mass 1000 kg moves along a straight horizontal road. The engine produces a driving force of 4000 N and the resistance to motion is 600 N.
Find the acceleration of the lorry.

Resultant force = 4000 − 600 = 3400 N
F = ma: 3400 = 1000 a

Answer: 3.4 m s^{-2}

2 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (−2, 6) and (−5, −1).
(a) Find the midpoint of AB.
(b) Find the line AB as ax + by + c = 0, with integer a, b, c.

(a) Midpoint = $(\frac{-2+(-5)}{2}, \frac{6+(-1)}{2}) = (-\frac{7}{2}, \frac{5}{2})$

(b) Gradient = $\frac{-7}{-3} = \frac{7}{3}$

$y - 6 = \frac{7}{3}(x + 2)$

Multiply out and collect terms: $7x - 3y + 32 = 0$

Answer: (a) $(-\frac{7}{2}, \frac{5}{2})$ (b) $7x - 3y + 32 = 0$

3 Finding the mean or standard deviation (All boards)

A journey takes X minutes, normal with s.d. 6.
Given P(X > 488.2) = 0.025, find the mean (3 s.f.).

P(X > 488.2) = 0.025, so P(X < 488.2) = 0.975

z = −1.960: $\frac{488.2 - \mu}{6} = -1.960$

$\mu = 488.2 + 1.960 \times 6 = 500$

Answer: $\mu = 500$

4 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A charity raises £150 in week 1. In each following week it raises £15 more than in the previous week.
(a) Find the amount raised in week 13.
(b) Find the total for the first 24 weeks.

(a) Arithmetic sequence with a = 150, d = 15
$u_{13} = 150 + 12 \times 15 = £330$

(b) $S_{24} = \frac{24}{2}(2 \times 150 + 23 \times 15) = £7{,}740$

Answer: (a) £330 (b) £7,740

5 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), find an approximation of the form $1 + p\theta + q\theta^2$ for
$\sin 5\theta + \cos 3\theta + 2\tan\theta$.

$\sin 5\theta \approx 5\theta$, $\cos 3\theta \approx 1 - \frac{9\theta^2}{2}$, $\tan\theta \approx \theta$

Sum $\approx 1 + 5\theta - \frac{9\theta^2}{2} + 2\theta$

$\approx 1 + 7\theta - \frac{9\theta^2}{2}$

Answer: $p = 7$, $q = -\frac{9}{2}$

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6 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 50t - 5t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 50 - 10t$

Maximum speed when $a = 0$: $t = 50 \div 10 = 5$

$v = 50 \times 5 - 5 \times 5^2$

Answer: 125 m s^{-1} at t = 5 s

7 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{6}} (3\cos^2 3x - 1)\,dx$.

Use $\cos^2 3x = \frac{1}{2}(1 + \cos 6x)$

$\int (3\cos^2 3x - 1)\,dx = \frac{x}{2} + \frac{\sin 6x}{4}$

$= \left[\frac{x}{2} + \frac{\sin 6x}{4}\right]_0^{\frac{\pi}{6}}$

$= \frac{\pi}{12}$

Answer: $\frac{\pi}{12}$

8 Two independent binomials (All boards)

X ~ B(11, 0.3) and Y ~ B(8, 0.1) are independent.
Find P(X = 4 and Y ≤ 1), to 3 significant figures.

P(X = 4) = 0.2201

P(Y ≤ 1) = 0.8131

Independent, so multiply: 0.2201 × 0.8131 = 0.179

Answer: 0.179

9 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(7, −3) and B(−4, −6), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:

$49 + (h + 3)^2 = 16 + (h + 6)^2$

Expanding and cancelling h^2: $6h + 58 = 12h + 52$

$h = 1$

$r^2 = CA^2 = 65$

Answer: centre (0, 1), $x^2 + (y - 1)^2 = 65$

10 Equations using sin²x + cos²x = 1 (All boards)

Solve $\sin x + 6\cos^2 x - 4 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2 x = 1 - \sin^2 x$: $\sin x + 2 - 6\sin^2 x = 0$, i.e. $6\sin^2 x - \sin x - 2 = 0$

$(3\sin x - 2)(2\sin x + 1) = 0$, so $\sin x = \frac{2}{3}$ or $-\frac{1}{2}$

$\sin x = \frac{2}{3}$: $x = 41.8°, 138.2°$

$\sin x = -\frac{1}{2}$: $x = 210.0°, 330.0°$

Answer: x = 41.8°, 138.2°, 210.0°, 330.0°