Day 80 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Basic trig equations (degrees) (All boards)

Solve cos (x + 50°) = 0.55 for 0 ≤ x ≤ 360°, giving answers to 1 d.p.

Let θ = x + 50°, so cos θ = 0.55 with θ in range 50° to 410°
θ = 56.63°, 303.37°
x = 6.6°, 253.4°
Answer: x = 6.6°, 253.4°

2 Resultant of perpendicular forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two forces of magnitude 18 N and 24 N act at right angles on a particle.
Find the magnitude of the resultant, and the angle it makes with the 18 N force, to 1 d.p.

Magnitude = $\sqrt{(18^2 + 24^2)} = \sqrt{900}$
$\tan \alpha = 24 \div 18$
Answer: 30 N at 53.1°

3 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence has 7th term 33 and 11th term 49.
(a) Find the first term and the common difference.
(b) Find the sum of the first 30 terms.

a + 6d = 33 and a + 10d = 49
Subtract: 4d = 16, so d = 4 and a = 9

$S_{30} = \frac{30}{2}(2 \times 9 + 29 \times 4) = 2010$

Answer: (a) a = 9, d = 4 (b) 2010

4 Venn diagram with independent events (All boards)

The Venn diagram shows probabilities for events A and B, which are independent.
Find the values of x and y.

P(A) = 0.21 + 0.09 = 0.3 and P(A ∩ B) = 0.09
Independent: P(A ∩ B) = P(A)P(B), so P(B) = 0.09 ÷ 0.3 = 0.3
x = P(B) − P(A ∩ B) = 0.3 − 0.09 = 0.21
y = 1 − (0.21 + 0.09 + 0.21) = 0.49
Answer: x = 0.21, y = 0.49

5 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 13 cm and arc length 14.3 cm.
Find (a) the angle of the sector in radians, (b) the area of the sector.

$s = r\theta$, so $\theta = \frac{14.3}{13} = 1.1$

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 13^2 \times 1.1 = 92.95$

Answer: (a) 1.1 rad (b) 92.95 cm^2

6 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(−5, −6) and B(−7, −2), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:
$25 + (h + 6)^2 = 49 + (h + 2)^2$
Expanding and cancelling h^2: 12h + 61 = 4h + 53
h = −1
$r^2 = CA^2 = 50$
Answer: centre (0, −1), $x^2 + (y + 1)^2 = 50$

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7 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected horizontally at 294 m s^{-1} from a point high above the ground.
Take g = 9.8 m s^{-2}.
Find the speed and the direction of motion 40 s later, to 1 d.p. below the horizontal.

Vertical speed = 9.8 × 40 = 392
Speed = $\sqrt{(294^2 + 392^2)} = 490$
tan α = 392 ÷ 294, α = 53.1°
Answer: 490 m s^{-1} at 53.1° below horizontal

8 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $x^2 - 12x + 46$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the minimum point of $y = x^2 - 12x + 46$.

$x^2 - 12x = (x - 6)^2 - 36$
So $x^2 - 12x + 46 = (x - 6)^2 + 10$
$(x - 6)^2 \geq 0$ with equality at x = 6, so the minimum is 10 at x = 6.
Answer: (a) $(x - 6)^2 + 10$ (b) (6, 10)

9 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
$x = 2t^2$, $y = 4t$.
Find the gradient at t = −2 and the equation of the normal there.

$\frac{dx}{dt} = 4t$, $\frac{dy}{dt} = 4$

Gradient = $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{1}{2}$ at t = −2

Point: (8, −8); normal gradient = 2
y + 8 = 2(x − 8), so y = 2x − 24
Answer: gradient $-\frac{1}{2}$; normal y = 2x − 24

10 Binomial two-tailed test (All boards)

A factory says a proportion 0.2 of its parts are defective. In a sample of 40 parts, 2 are defective.
Test, at the 5% significance level, whether the proportion is different from 0.2.

H_0: p = 0.2, H_1: p ≠ 0.2. Under H_0, X ~ B(40, 0.2)
P(X ≤ 2) = 0.0079
Two-tailed: compare with 0.025. 0.0079 < 0.025, so reject H_0
There is sufficient evidence that the proportion is different from 0.2
Answer: reject H_0: sufficient evidence that the proportion is different from 0.2