Day 8 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = 3t^2 + 8t + 1$ m s^{-1}.
Find the distance moved between t = 1 and t = 5. (The particle does not change direction.)

$s = \int_1^5 (3t^2 + 8t + 1)\,dt$

$= [\,t^3 + 4t^2 + t\,]$ from t = 1 to t = 5

$= 230 - 6$

Answer: 224 m

2 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence has 10th term 26 and 12th term 30.
(a) Find the first term and the common difference.
(b) Find the sum of the first 20 terms.

$a + 9d = 26$ and $a + 11d = 30$

Subtract: $2d = 4$, so $d = 2$ and $a = 8$

$S_{20} = \frac{20}{2}(2 \times 8 + 19 \times 2) = 540$

Answer: (a) a = 8, d = 2 (b) 540

3 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 26 students.
Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 26$, $\Sigma fx = 74$, $\Sigma fx^2 = 238$

Mean $= \frac{74}{26} = 2.85$

Variance $= \frac{238}{26} - (2.8462)^2 = 1.053$

Standard deviation = 1.03

Answer: mean 2.85, standard deviation 1.03

4 Product rule (All boards)

Given that $y = x^4 \sin 4x$,
find $\frac{dy}{dx}$, and its value when $x = \frac{\pi}{8}$.

Product rule with $u = x^4$, $v = \sin 4x$

$u' = 4x^3$, $v' = 4\cos 4x$

$\frac{dy}{dx} = u'v + uv' = 4x^3(x\cos 4x + \sin 4x)$

When $x = \frac{\pi}{8}$: $\frac{dy}{dx} = \frac{\pi^3}{128}$

Answer: $\frac{dy}{dx} = 4x^3(x\cos 4x + \sin 4x)$; value $\frac{\pi^3}{128}$

5 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 3t + 3$, $y = -t^2 - 4t - 1$.
The line $y = -2x + 5$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $-t^2 - 4t - 1 = -6t - 1$

$-t^2 + 2t = 0$, so $-(t - 2)t = 0$

t = 0 or t = 2, giving the points (3, −1) and (9, −13)

Answer: (3, −1) and (9, −13)

6 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = \frac{\ln x}{x}$, and determine its nature.

$\frac{dy}{dx} = \frac{1 - \ln x}{x^2} = 0$

$x = e$

$\frac{d^2y}{dx^2} = \frac{2 \ln x - 3}{x^3}$

At $x = e$: $\frac{d^2y}{dx^2} = -\frac{1}{e^3} < 0$, so a maximum

Answer: $(e, \frac{1}{e})$ maximum

7 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 10 kg lies on a rough horizontal table (coefficient of friction 0.2). It is joined by a light string over a smooth pulley to a particle of mass 5 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.2 \times 10\,g = 10\,a$

Hanging: $5\,g - T = 5\,a$

Add: $5\,g - 2\,g = 15\,a$

$a = 1.96$

$T = 5(9.8 - 1.96)$

Answer: $a = 1.96$ m s^{-2}; $T = 39.2$ N

8 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-3x^2 + 25x - 56}{(x-4)(x-5)^2}$ in partial fractions.

$\frac{-3x^2 + 25x - 56}{(x-4)(x-5)^2} \equiv \frac{A}{(x-4)} + \frac{B}{(x-5)} + \frac{C}{(x-5)^2}$

$-3x^2 + 25x - 56 \equiv A(x-5)^2 + B(x-4)(x-5) + C(x-4)$

$x = 4$ gives $A = -4$; $x = 5$ gives $C = -6$

Coefficients of x^2: $-3 = A + B$, so $B = 1$

Answer: $-\frac{4}{(x-4)} + \frac{1}{(x-5)} - \frac{6}{(x-5)^2}$

9 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.25. In a sample of 50 students, 17 pass first time.

Test, at the 5% significance level, whether the probability is different from 0.25.

H_0: $p = 0.25$, H_1: $p \neq 0.25$. Under H_0, $X \sim B(50, 0.25)$

$P(X \geq 17) = 1 - P(X \leq 16) = 0.0983$

Two-tailed: compare with 0.025. $0.0983 > 0.025$, so do not reject H_0

There is insufficient evidence that the probability is different from 0.25

Answer: do not reject H_0: insufficient evidence that the probability is different from 0.25

10 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 6 cm and angle 120°.

Find the perimeter of the sector, in terms of π.

$\theta = \frac{2\pi}{3}$ radians

Arc length $= r\theta = 4\pi$ cm

Perimeter $= 2r +$ arc $= 12 + 4\pi$ cm

Answer: $12 + 4\pi$ cm