Day 79 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} -2 \\ -6 \\ 9 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

$|\mathbf{a}| = \sqrt{2^2 + 6^2 + 9^2} = \sqrt{121} = 11$

Unit vector $= \frac{1}{11}\mathbf{a}$

Answer: $|\mathbf{a}| = 11$, unit vector $\frac{1}{11}\begin{pmatrix} -2 \\ -6 \\ 9 \end{pmatrix}$

2 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $(\frac{81}{16})^{-\frac{1}{2}}$.

$\frac{81}{16} = (\frac{3}{2})^4$, so the expression $= (\frac{3}{2})^{-2}$

$= \frac{4}{9}$

Answer: $\frac{4}{9}$

3 Fixed-point iteration (All boards)

(a) Show that $x^2 - 3x - 4 = 0$ can be written as $x = \sqrt{3x + 4}$.

(b) Use $x_{n+1} = \sqrt{3x_n + 4}$ with $x_0 = 3$ to find x_1, x_2, x_3 to 4 d.p.

(a) $x^2 = 3x + 4$, so $x = \sqrt{3x + 4}$

(b) $x_1 = 3.6056$

$x_2 = 3.8492$

$x_3 = 3.9431$

Answer: $x_1 = 3.6056$, $x_2 = 3.8492$, $x_3 = 3.9431$

4 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the marks in a mock exam for two groups.

(a) Find the interquartile range of each group.

(b) Compare the two distributions.

(a) Year 12: $55 - 35 = 20$. Year 13: $40 - 25 = 15$

(b) Medians: 40 (Year 12) and 30 (Year 13), so Year 12 has the higher median

IQRs: Year 12 has the larger IQR, so its scores are more spread out

Answer: (a) 20 and 15 (b) Year 12 has the higher median; Year 12 is more spread out (larger IQR), Year 13 more consisten

5 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|3x + 4| = |-x - 2|$.

Either $3x + 4 = -x - 2$ or $3x + 4 = -(-x - 2)$

$3x + 4 = -x - 2$ gives $x = -\frac{3}{2}$

$3x + 4 = -(-x - 2)$ gives $x = -1$

Answer: $x = -\frac{3}{2}$ and $x = -1$

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6 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (3t^2 + 5)\mathbf{i} + (5t - 20)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0
$5t - 20 = 0$, so $t = 4$
$\mathbf{v} = (3 \times 4^2 + 5)\mathbf{i} = 53\mathbf{i}$
Answer: t = 4 s, speed 53 m s^{-1}

7 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-5x - 17}{(x + 3)(x + 4)}$ in partial fractions.

(b) Hence evaluate $\int_0^1 \frac{-5x - 17}{(x + 3)(x + 4)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-5x - 17}{(x + 3)(x + 4)} \equiv \frac{A}{x + 3} + \frac{B}{x + 4}$, so $-5x - 17 \equiv A(x + 4) + B(x + 3)$
$x = -3$: $A = -2$; $x = -4$: $B = -3$
(b) $\int_0^1 \left(\frac{-2}{x + 3} + \frac{-3}{x + 4}\right) dx = [-2\ln(x + 3) - 3\ln(x + 4)]_0^1$
$= -2\ln\frac{4}{3} - 3\ln\frac{5}{4} = \ln\left(\frac{36}{125}\right)$
Answer: (a) A = −2, B = −3 (b) $\ln\left(\frac{36}{125}\right)$

8 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which
$4x + 4 \leq -3$ and $x^2 - 2x - 15 \geq 0$.

$4x + 4 \leq -3$ gives $x \leq -\frac{7}{4}$
$x^2 - 2x - 15 \geq 0$: $(x + 3)(x - 5) \geq 0$ gives $x \leq -3$ or $x \geq 5$
Both must hold: $x \leq -3$
Answer: $x \leq -3$

9 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
P(X < 34) = 0.0228 and P(X > 58) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.999$ and $z_2 = 0.9998$
$34 = \mu + (-1.999)\sigma$ and $58 = \mu + 0.9998\sigma$
Subtract: $24 = 2.999\,\sigma$, so $\sigma = 8.00$
$\mu = 34 - (-1.999)(8.003) = 50.0$
Answer: $\mu = 50.0$, $\sigma = 8.00$

10 Beam on two supports: reactions (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform beam AB of length 5 m and mass 50 kg rests horizontally on supports at C and D, where AC = 1 m and BD = 1.5 m.
A particle of mass 10 kg is placed on the beam 1.5 m from B. Take g = 9.8 m s^{-2}.
Find the reactions at C and D.

Moments about C: $R_D \times 2.5 = 50\text{ g} \times 1.5 + 10\text{ g} \times 2.5$
$R_D = 980 \div 2.5$
Resolve vertically: $R_C + R_D = 60\text{ g}$
Answer: $R_C = 196$ N; $R_D = 392$ N