Day 78 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Binomial probabilities (All boards)

X ~ B(10, 0.7). Find, to 3 significant figures,
(a) P(X ≥ 7)
(b) P(X > 6)

(a) P(X ≥ 7) = 1 − P(X ≤ 6)
(a) = 0.650
(b) P(X > 6) = 1 − P(X ≤ 6)
(b) = 0.650
Answer: (a) 0.650 (b) 0.650

2 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sprinter moves in a straight line with constant acceleration. Its initial speed is 10 m s^{-1}, its final speed is 22 m s^{-1} and its acceleration is 1 m s^{-2}.
Find the time taken.

v = u + at
22 = 10 + 1 t
1 t = 12
Answer: 12 s

3 Exponential growth and decay models (All boards)

A car's value, £V, after t years is given by $V = 12000 \times 0.9^t$.
Find (a) the value after 3 years, (b) when it first falls below £3000, to 3 s.f.

(a) $V = 12000 \times 0.9^3 = 8748$
(b) $0.9^t < 0.25$, so $t \ln 0.9 < \ln 0.25$

$\ln 0.9 < 0$, so the inequality reverses: $t > \frac{\ln 0.25}{\ln 0.9} = 13.16$

Answer: (a) £8748 (b) t > 13.2 years

4 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 3 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{3}{5}$. Find P and Q.

Resolve horizontally: $P \cos\theta = 3$, so $P \times \frac{3}{5} = 3$

P = 5

Resolve vertically: $Q = P \sin\theta = 5 \times \frac{4}{5}$

Answer: P = 5 N; Q = 4 N

5 Area under a curve (All boards)

The region R is bounded by the axes, the line x = ln 2
and the curve $y = 2e^x + 1$. Find the exact area of R.

Area = $\int_0^{\ln 2} (2e^x + 1)\,dx = [x + 2e^x]_0^{\ln 2}$
= ln 2 + 2
Answer: ln 2 + 2

6 Binomial one-tailed test (All boards)

A manufacturer claims that a proportion 0.4 of its sweets are red. A sample of 25 sweets has 7 red.
Test, at the 10% level, whether the true proportion is lower than claimed.

H_0: p = 0.4, H_1: p < 0.4. Let X ~ B(25, 0.4) under H_0
P(X ≤ 7) = 0.1536
0.1536 > 0.1, so do not reject H_0
There is insufficient evidence that the proportion is lower than claimed
Answer: do not reject H_0: insufficient evidence that the proportion is lower

7 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{12}{3+\sqrt{6}}$ in the form $p + q\sqrt{6}$, where p and q are integers.

Multiply top and bottom by the conjugate $(3 - \sqrt{6})$:
Denominator: $(3 + \sqrt{6})(3 - \sqrt{6}) = 3^2 - 1^2 \times 6 = 3$
Numerator: $(12)(3 - \sqrt{6}) = 36 - 12\sqrt{6}$
So the fraction = $\frac{36 - 12\sqrt{6}}{3} = 12 - 4\sqrt{6}$
Answer: $12 - 4\sqrt{6}$

8 Quotient rule (All boards)

Find the x-coordinate(s) of the stationary point(s) of the curve $y = \frac{x}{x^2 + 4}$.

$\frac{dy}{dx} = \frac{4 - x^2}{(x^2 + 4)^2}$
Stationary points where $\frac{dy}{dx} = 0$, i.e. the numerator is zero
Numerator: $-(x - 2)(x + 2) = 0$
x = −2 or x = 2
Answer: x = −2, x = 2

9 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + 5n + 9$ is odd for all integers n.

$n^2 + 5n = n(n + 5)$
As 5 is odd, n and n + 5 have opposite parity,
so one of them is even and the product n(n + 5) is even.
Even + 9 = odd, so $n^2 + 5n + 9$ is odd.
Answer: n(n + 5) is even, so the expression is odd

10 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = t - 1$, $y = 2t^2 + 4t - 1$.
The line $y = -2x - 7$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $2t^2 + 4t - 1 = -2t - 5$
$2t^2 + 6t + 4 = 0$, so $2(t + 1)(t + 2) = 0$
t = −2 or t = −1, giving the points (−3, −1) and (−2, −3)
Answer: (−3, −1) and (−2, −3)