Day 77 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-4, -5)$ and $(-5, 2)$.
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{-4 + (-5)}{2}, \frac{-5 + 2}{2}\right) = \left(-\frac{9}{2}, -\frac{3}{2}\right)$

(b) Gradient $= \frac{7}{-1} = -7$

$y + 5 = -7(x + 4)$
Multiply out and collect terms: $7x + y + 33 = 0$

Answer: (a) $\left(-\frac{9}{2}, -\frac{3}{2}\right)$ (b) $7x + y + 33 = 0$

2 Inverse normal (All boards)

The length X cm of a metal rod is normal with mean 170, s.d. 4.
Find k (3 s.f.): (a) $P(X < k) = 0.05$,
(b) $P(X > k) = 0.2$.

(a) $z = -1.645$, so $k = 170 + (-1.645) \times 4 = 163$
(b) $P(X < k) = 0.8$, $z = 0.8416$, so $k = 170 + 0.8416 \times 4 = 173$
Answer: (a) 163 (b) 173

3 Product rule (All boards)

Given that $y = e^{-2x} \cos x$,
find $\frac{dy}{dx}$, and its value when $x = \frac{\pi}{2}$.

Product rule with $u = e^{-2x}$, $v = \cos x$
$u' = -2e^{-2x}$, $v' = -\sin x$
$\frac{dy}{dx} = u'v + uv' = (-\sin x - 2\cos x)e^{-2x}$

When $x = \frac{\pi}{2}$: $\frac{dy}{dx} = -\frac{1}{e^{\pi}}$

Answer: $\frac{dy}{dx} = (-\sin x - 2\cos x)e^{-2x}$; value $-\frac{1}{e^{\pi}}$

4 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (3, 5), (4, 4) and (6, 6).
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB $= \frac{1}{-1} = -1$

Gradient BC $= \frac{2}{2} = 1$

$(-1) \times 1 = -1$, so AB and BC are perpendicular.
(b) AB $= \sqrt{2}$ and BC $= 2\sqrt{2}$

Area $= \frac{1}{2} \times AB \times BC = 2$

Answer: (b) area = 2

5 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 4 s, (b) the distance between the stations.

(a) a = gradient = 15 ÷ 4
(b) Distance = area of trapezium = ½ × (21 + 12) × 15
Answer: (a) 3.75 m s^{-2} (b) 247.5 m

6 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations

$x = t^3$, $y = t^2 + 2t$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the tangent at t = 3.

$\frac{dx}{dt} = 3t^2$, $\frac{dy}{dt} = 2t + 2$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2(t + 1)}{3t^2}$; at t = 3 it is $\frac{8}{27}$

Point: (27, 15)

$y - 15 = \frac{8}{27}(x - 27)$, so $y = \frac{8}{27}x + 7$

Answer: $\frac{dy}{dx} = \frac{2(t + 1)}{3t^2}$; tangent $y = \frac{8}{27}x + 7$

7 Pulling at an angle: constant speed on rough surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A suitcase of mass 3 kg is dragged at constant speed along rough horizontal ground by a rope inclined at angle α above the horizontal, where $\cos \alpha = \frac{3}{5}$.

The coefficient of friction is 0.5. Take g = 9.8 m s^{-2}. Find the tension in the rope.

Vertical: $R + T \sin \alpha = mg$, so $R = 29.4 - \frac{4}{5}T$

Horizontal: $T \cos \alpha = \mu R$: $\frac{3}{5}T = 0.5(29.4 - \frac{4}{5}T)$

$T(\cos \alpha + \mu \sin \alpha) = \mu mg$: $T \times 1 = 0.5 \times 3 \times 9.8$ (since $\cos \alpha + \mu \sin \alpha = 1$)

Answer: 14.7 N

8 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf $f(x) = kx^4$ for $0 \le x \le 6$, and 0 otherwise.

(a) Find the value of k.

(b) Find P(X > 4). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^5/5]$ from 0 to 6 $= k \times \frac{6^5}{5} = 1$, so $k = \frac{5}{7776}$

(b) $P(X > 4) = \int_4^6 \frac{5}{7776} x^4\,dx = 1 - \frac{4^5}{6^5} = \frac{211}{243}$

(c) $E(X) = \int_0^6 x\,f(x)\,dx = \frac{5}{7776} \times \frac{6^6}{6} = 5$

Answer: (a) $k = \frac{5}{7776}$ (b) $\frac{211}{243}$ (c) 5

9 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=8}^{20}(2r + 2)$.

There are $20 - 8 + 1 = 13$ terms

First term = 18, last term = 42

$S = \frac{13}{2}(18 + 42) = 390$

Answer: 390

10 Double angle equations (All boards)

Solve $\cos 2x + 3 \sin x = 2$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $1 - 2\sin^2 x + 3 \sin x - 2 = 0$

$(2 \sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2}$: $x = 30.0°, 150.0°$

$\sin x = 1$: $x = 90.0°$

Answer: $x = 30.0°, 90.0°, 150.0°$