**Day 76** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $0 < 2x - 5 \leq 13$.

Subtract $-5$ from each part: $5 < 2x \leq 18$

Divide each part by 2: $\frac{5}{2} < x \leq 9$

**Answer:** $\mathbf{\frac{5}{2} < x \leq 9}$

## 2 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lorry of mass 1000 kg moves along a straight horizontal road. The engine produces a driving force of 3500 N and the resistance to motion is 400 N.
Find the acceleration of the lorry.

Resultant force = $3500 - 400 = 3100$ N
F = ma: $3100 = 1000\,a$

**Answer:** $\mathbf{3.1\ m\ s^{-2}}$

## 3 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = 1$ and $\theta$ is acute,
find the exact value of $\operatorname{cosec}\theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
$\tan\theta = 1$, so $\sec^2\theta = 1 + 1 = 2$
$\cot\theta = 1$, so $\operatorname{cosec}^2\theta = 2$ and $\operatorname{cosec}\theta = \sqrt{2}$

**Answer:** $\mathbf{\sqrt{2}}$

## 4 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 5 kg is acted on by forces $(2\mathbf{i} + 5\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(3\mathbf{i} + 4\mathbf{j})$ m s$^{-2}$.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(2\mathbf{i} + 5\mathbf{j}) + \mathbf{P} = 5(3\mathbf{i} + 4\mathbf{j}) = (15\mathbf{i} + 20\mathbf{j})$
$\mathbf{P} = 13\mathbf{i} + 15\mathbf{j}$
$|\mathbf{P}| = \sqrt{(13^2 + 15^2)}$

**Answer:** $\mathbf{P = (13i + 15j)}$ **N;** $\mathbf{|P| = 19.8}$ **N**

## 5 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = \frac{1}{x}$, the x-axis
and the lines $x = 1$ and $x = 3$.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_1^3 y^2\,dx = \pi \int_1^3 \frac{1}{x^2}\,dx$

$= \pi\left[-\frac{1}{x}\right]_1^3$

$= \frac{2\pi}{3}$

**Answer:** $\mathbf{\frac{2\pi}{3}}$

## 6 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of $x^3$ in the expansion of
$(2 + x)(1 - 3x)^4$.

$(1 - 3x)^4$ has $x^2$ coefficient 54 and $x^3$ coefficient $-108$
$x^3$ term of the product $= 2 \times (-108) + 1 \times 54$
$= -162$

**Answer:** $\mathbf{-162}$

## 7 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} + 2e^x - 8 = 0$,
giving any answers as exact values.

Let $y = e^x$: $y^2 + 2y - 8 = 0$
$(y + 4)(y - 2) = 0$
$e^x > 0$, so $y = -4$ is rejected; $e^x = 2$
$x = \ln 2$
**Answer: $x = \ln 2$**

## 8 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $3^{2x} - 12 \times 3^x + 27 = 0$.

Let $y = 3^x$. Then $3^{2x} = y^2$, so $y^2 - 12y + 27 = 0$
$(y - 9)(y - 3) = 0$
$y = 9$ or $y = 3$
$3^x = 9$ gives $x = 2$; $3^x = 3$ gives $x = 1$.
**Answer: $x = 1$ or $x = 2$**

## 9 Binomial mean and variance (All boards)

X has a binomial distribution with mean 4.8 and variance 1.92.
(a) Find n and p.
(b) Find P(X = 4) to 3 significant figures.

(a) $np = 4.8$ and $np(1 - p) = 1.92$
$1 - p = 1.92 \div 4.8 = 0.4$, so $p = 0.6$ and $n = 4.8 \div 0.6 = 8$
(b) $P(X = 4) = {}^8C_4 \times 0.6^4 \times 0.4^4 = 0.232$
**Answer: (a) $n = 8$, $p = 0.6$ (b) 0.232**

## 10 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
$P(X < 32) = 0.0668$ and $P(X > 62) = 0.1587$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.500$ and $z_2 = 0.9998$
$32 = \mu + (-1.500)\sigma$ and $62 = \mu + 0.9998\sigma$
Subtract: $30 = 2.500\,\sigma$, so $\sigma = 12.0$
$\mu = 32 - (-1.500)(12.00) = 50.0$
**Answer: $\mu = 50.0$, $\sigma = 12.0$**