**Day 75** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Chain rule (All boards)

Given that $y = \ln(3x + 7)$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = 3$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = \frac{3}{3x + 7}$

When $x = 3$: gradient $= \frac{3}{16}$

**Answer: $\frac{dy}{dx} = \frac{3}{3x + 7}$; gradient $\frac{3}{16}$**

## 2 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A person of mass 80 kg stands on the floor of a lift. The lift accelerates upwards at 2 m $s^{-2}$.

Take g = 9.8 m $s^{-2}$.

Find the normal reaction between the person and the floor.

Resolve vertically (up positive): $R - 80 \times 9.8 = 80 \times 2$

$R = 80(9.8 + 2)$

**Answer: 944 N**

## 3 Solving exponential equations using logarithms (All boards)

Solve $5^{3x - 3} = 39$, giving your answer to 3 s.f.

Take logs: $(3x - 3) \ln 5 = \ln 39$

$3x - 3 = \frac{\ln 39}{\ln 5} = 2.2763$

$x = 1.759$

**Answer: x = 1.76**

## 4 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 13 pairs of values, $\bar{x} = 8$, $\bar{y} = 30$, $S_{xx} = 50$ and $S_{xy} = 125$.

(a) Find the regression line of y on x in the form $y = a + bx$.

(b) Estimate y when x = 11.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{125}{50} = 2.5$

$a = \bar{y} - b\bar{x} = 30 - 2.5 \times 8 = 10$

$y = 10 + 2.5x$

(b) $y = 10 + 2.5 \times 11 = 37.5$

**Answer: y = 10 + 2.5x; y = 37.5**

## 5 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 + 3x - 1$.

C is stretched by scale factor 2 parallel to the y-axis.

Find its new equation.

Stretch parallel to the y-axis: multiply y by the scale factor.

When stretched by scale factor 2 parallel to the y-axis: $y = 2x^2 + 6x - 2$

**Answer: $y = 2x^2 + 6x - 2$**

## 6 Car towing a trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 1500 kg tows a trailer of mass 500 kg along a straight horizontal road by a light horizontal tow bar. The resistances are 500 N on the car and 100 N on the trailer.

The engine's driving force is 2800 N. Find the acceleration and the tension in the tow bar.

Whole system: $2800 - 500 - 100 = 2000\,a$

$a = 1.1$

Trailer: $T - 100 = 500 \times 1.1$

**Answer: a = 1.1 m $s^{-2}$; T = 650 N**

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## 7 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{12 + 5\sqrt{6}}{\sqrt{6}}$ in the form $p + q\sqrt{6}$, where p and q are integers.

Multiply top and bottom by $\sqrt{6}$:

$\frac{(12 + 5\sqrt{6})\sqrt{6}}{6} = \frac{12\sqrt{6} + 30}{6}$

$= 5 + 2\sqrt{6}$

**Answer: $5 + 2\sqrt{6}$**

## 8 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} + e^{x} - 6 = 0$,
giving any answers as exact values.

Let $y = e^{x}$: $y^2 + y - 6 = 0$

$(y + 3)(y - 2) = 0$

$e^{x} > 0$, so $y = -3$ is rejected; $e^{x} = 2$

$x = \ln 2$

**Answer: $x = \ln 2$**

## 9 Binomial: smallest n (All boards)

Given that each trap catches a rabbit independently with probability 0.1, find the least number of traps needed for the probability of at least one success to exceed 0.999.

P(at least one) $= 1 - 0.9^{n} > 0.999$

$0.9^{n} < 0.001$

$n > \frac{\log 0.001}{\log 0.9} = 65.56$

**Answer: $n = 66$**

## 10 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x - 11$ meets the circle $(x + 3)^2 + (y + 1)^2 = 169$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -x - 11$: $(x + 3)^2 + (-x - 10)^2 = 169$

$x^2 + 13x - 30 = 0$

$(x - 2)(x + 15) = 0$, so $x = -15$ or $x = 2$

$y = -x - 11$: A = $(-15, 4)$, B = $(2, -13)$

$AB^2 = 17^2 + (-17)^2 = 578$

**Answer: A$(-15, 4)$, B$(2, -13)$, AB $= 17\sqrt{2}$**