Day 74 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moves in a straight line with constant acceleration. Its initial speed is 2 m s^{-1}, its acceleration is 2 m s^{-2} and the motion lasts 12 s.
Find the final speed.

$v = u + at$
$v = 2 + 2 \times 12$
Answer: 26 m s^{-1}

2 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 3t^2$, $y = 4t$, $t \in \mathbb{R}$.
Find a Cartesian equation of the curve.

From $y = 4t$: $t = \frac{y}{4}$

$x = 3(\frac{y}{4})^2 = \frac{3y^2}{16}$

So $y^2 = \frac{16}{3}x$

Answer: $y^2 = \frac{16}{3}x$

3 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the test scores for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Class A: $55 - 30 = 25$. Class B: $40 - 20 = 20$
(b) Medians: 40 (Class A) and 35 (Class B), so Class A has the higher median
IQRs: Class A has the larger IQR, so its scores are more spread out
Answer: (a) 25 and 20 (b) Class A has the higher median; Class A is more spread out (larger IQR), Class B more consistent

4 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $xe^y - 2y = 3$.
Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at (3, 0).

Differentiate with respect to x: $x\frac{dy}{dx}e^y + e^y - 2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{e^y}{xe^y - 2}$
At (3, 0): gradient = -1
Answer: $\frac{dy}{dx} = -\frac{e^y}{xe^y - 2}$; gradient -1

5 Work-energy: sliding to rest (CIE)

A block of mass 3 kg is projected along a rough horizontal surface with speed 7 m s^{-1} and comes to rest after sliding 10 m.
Take g = 9.8 m s^{-2}.
Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE
$\mu \times 3 \times 9.8 \times 10 = \frac{1}{2} \times 3 \times 7^2$
$294\mu = 73.5$
Answer: 0.25

6 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve y = f(x), where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

The curve touches the x-axis at $x = -3$ (repeated root) and crosses at $x = 2$

So $y = k(x + 3)^2(x - 2)$

The curve passes through $(0, -18)$: $k \times 9 \times (-2) = -18$

So $k = 1$

Answer: $f(x) = (x + 3)^2(x - 2)$

7 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = e^{3x}$, the x-axis and the lines $x = 0$ and $x = 1$.

R is rotated through 360° about the x-axis. Find the exact volume generated.

$$V = \pi \int_0^1 y^2 \, dx = \pi \int_0^1 e^{6x} \, dx$$

$$= \pi\left[\frac{e^{6x}}{6}\right]_0^1$$

$$= -\frac{\pi(1 - e^6)}{6}$$

Answer: $-\frac{\pi(1 - e^6)}{6}$

8 Binomial: smallest n (All boards)

Given that each lottery ticket wins a prize independently with probability 0.1, find the least number of tickets needed for the probability of at least one success to exceed 0.99.

$P(\text{at least one}) = 1 - 0.9^n > 0.99$

$0.9^n < 0.01$

$n > \frac{\log 0.01}{\log 0.9} = 43.71$

Answer: n = 44

9 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} -1 \\ 5 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 5 \\ -1 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a} \cdot \mathbf{b} = (-1)(-1) + (5)(5) + (-3)(-1) = 29$

$|\mathbf{a}| = \sqrt{35}$, $|\mathbf{b}| = \sqrt{27}$

$\cos\theta = \frac{29}{\sqrt{35}\sqrt{27}} = 0.9434$

$\theta = 19.4°$

Answer: 19.4°

10 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 - 6x^2 + kx + 48$.

Given that $(x - 2)$ is a factor of f(x),

(a) find the value of k,

(b) factorise f(x) completely.

$f(2) = 0$: $16 - 24 + 2k + 48 = 0$

$k = -20$

$f(x) = (x - 2)(2x^2 - 2x - 24)$

$= 2(x - 4)(x - 2)(x + 3)$

Answer: (a) $k = -20$ (b) $f(x) = 2(x - 4)(x - 2)(x + 3)$