Day 73 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Standard integrals (definite) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{8}} (5\sin 4x + 2\cos 4x)\,dx$.

$\int (5\sin 4x + 2\cos 4x)\,dx = \frac{\sin 4x}{2} - \frac{5\cos 4x}{4}$

$= [\frac{\sin 4x}{2} - \frac{5\cos 4x}{4}]_0^{\frac{\pi}{8}}$

$= \frac{7}{4}$

Answer: $\frac{7}{4}$

2 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre $(-5, 2)$ and passes through the point $(-3, 1)$.
Find an equation of the circle.

$r^2 = (-3 - (-5))^2 + (1 - 2)^2$
$= 4 + 1 = 5$
Answer: $(x + 5)^2 + (y - 2)^2 = 5$

3 Locating roots: change of sign (All boards)

The function $f(x) = e^x - 9x$.
Show that $f(x) = 0$ has a root between $x = 3$ and $x = 3.5$.

$f(3) = -6.91$
$f(3.5) = 1.62$
f is continuous and changes sign, so there is a root in the interval
Answer: f(3) < 0 and f(3.5) > 0: sign change

4 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 12 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{4}{5}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 12$, so $P \times \frac{4}{5} = 12$

$P = 15$

Resolve vertically: $Q = P\sin\theta = 15 \times \frac{3}{5}$

Answer: P = 15 N; Q = 9 N

5 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 - 3x - 3$.
C is stretched by scale factor 3 parallel to the y-axis.
Find its new equation.

Stretch parallel to the y-axis: multiply y by the scale factor.
When stretched by scale factor 3 parallel to the y-axis: $y = 3x^2 - 9x - 9$
Answer: $y = 3x^2 - 9x - 9$

6 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves so that its position vector relative to O at time t s is $\mathbf{r} = (t^2 + t)\mathbf{i} + (3t^3 - 5t)\mathbf{j}$ metres.
Find the velocity when t = 1, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (2t + 1)\mathbf{i} + (9t^2 - 5)\mathbf{j}$
$t = 1$: $\mathbf{v} = (3)\mathbf{i} + (4)\mathbf{j}$
Speed $= \sqrt{(3^2 + 4^2)}$
Answer: v = (3)i + (4)j m s^{-1}; speed 5 m s^{-1}

7 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A chord AB subtends an angle of 1.2 radians at the centre O of a circle of radius 9 cm.
Find the perimeter of the minor segment, to 3 s.f.

Arc = $r\theta = 9 \times 1.2 = 10.8$

Chord AB = $2r \sin(\frac{\theta}{2}) = 10.16$

Perimeter = 10.8 + 10.16 = 20.96 cm

Answer: 21.0 cm

8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that $\sqrt{7}$ is irrational.

Assume $\sqrt{7} = \frac{a}{b}$, where a and b are integers with no common factor.

Then $a^2 = 7b^2$, so a^2 is a multiple of 7, so a is a multiple of 7.

Let $a = 7k$. Then $7^2k^2 = 7b^2$, so $b^2 = 7k^2$.

So b^2, and hence b, is a multiple of 7.

Then a and b have a common factor 7, a contradiction, so $\sqrt{7}$ is irrational.

Answer: $\sqrt{7}$ is irrational

9 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf $f(x) = kx^3$ for $0 \le x \le 4$, and 0 otherwise.
(a) Find the value of k.
(b) Find P(X > 3). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^4/4]$ from 0 to 4 $= k \times \frac{4^4}{4} = 1$, so $k = \frac{1}{64}$

(b) $P(X > 3) = \int_3^4 \frac{1}{64} x^3\,dx = 1 - \frac{3^4}{4^4} = \frac{175}{256}$

(c) $E(X) = \int_0^4 x\,f(x)\,dx = \frac{1}{64} \times \frac{4^5}{5} = \frac{16}{5}$

Answer: (a) $k = \frac{1}{64}$ (b) $\frac{175}{256}$ (c) $\frac{16}{5}$

10 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ab^x$, where a and b are constants.
A plot of $\log_{10} y$ against x is a straight line with intercept 0.602 and gradient 0.301.
Find a and b to 2 significant figures.

$\log_{10} y = \log_{10} a + x \log_{10} b$

$\log_{10} a = 0.602$, so $a = 10^{0.602} = 4.0$

$\log_{10} b = 0.301$, so $b = 10^{0.301} = 2.0$

Answer: a = 4.0, b = 2.0