Day 72 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic series is 7 and the common difference is 1.
The sum of the first n terms is 444. Find the value of n.

$S_n = \frac{n}{2}(2 \times 7 + (n - 1) \times 1) = 444$

$n^2 + 13n - 888 = 0$
$(n - 24)(n + 37) = 0$
n must be positive, so n = 24
Answer: n = 24

2 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$.
Find |a| and a unit vector in the direction of a.

$|\mathbf{a}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7$
Unit vector $= \frac{1}{7}\mathbf{a}$

Answer: |a| = 7, unit vector $\frac{1}{7}\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$

3 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 4x − 3 for $1 \leq x < 7$.
Find the range of f.

f is increasing, so the extreme values occur at the ends of the domain.
f(1) = 1 and f(7) = 25
x = 1 is included and x = 7 is excluded, so f = 1 is included and f = 25 is excluded.
Answer: $1 \leq f(x) < 25$

4 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 4 kg is pulled along rough horizontal ground by a horizontal force of 15.84 N. It accelerates at 2 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the coefficient of friction between the sledge and the ground.

R = mg = 4 × 9.8 = 39.2
Resultant: 15.84 − F = 4 × 2, so F = 7.84
F = μR: μ = 7.84 ÷ 39.2
Answer: 0.2

5 Finding the mean or standard deviation (All boards)

The length X cm of a metal rod is normal with mean 60.
Given P(X < 67.9) = 0.7, find the standard deviation (3 s.f.).

z = 0.5244, so $\frac{67.9 - 60}{\sigma} = 0.5244$

$\sigma = 7.9 \div 0.5244 = 15.1$
Answer: σ = 15.1

6 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A rocket is projected from ground level with initial velocity components 23 m s^{-1} horizontally and 49 m s^{-1} vertically upwards.
Take g = 9.8 m s^{-2}.
Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 49^2 - 2 \times 9.8 \times H$, $H = 2401 \div 19.6$
(b) At the top the vertical velocity is zero, so the speed is the horizontal component.
Answer: (a) 122.5 m (b) 23 m s^{-1}

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7 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(150, 0.4). Use a normal approximation to find P(X ≤ 56), to 3 significant figures.

Y ~ N(np, np(1 − p)) = N(60.00, 36.00), so σ = $\sqrt{36.00}$ = 6.000
Use a continuity correction.
P(X ≤ 56) ≈ P(Y < 56.5) = P(Z < −0.5833) = 0.280
Answer: 0.280

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line 3x + 4y = k is a tangent to the circle
$(x + 1)^2 + y^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (3, 4).
A unit step in this direction is (3, 4)/5, so the points of contact are
(−1, 0) ± 10 × (3, 4)/5 = (−1, 0) ± (6, 8)
T = (5, 8) or (−7, −8), and k = 3x + 4y gives:
k = 47 or k = −53
Answer: k = 47 or k = −53

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -10 \\ -4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: −10 + 3λ = 3 + (−2)μ
j: −4 + 1λ = 1 + (−1)μ
Solving i and j: λ = 3, μ = 2
k check: 7 + (−1)(3) = 4 and 6 + (−1)(2) = 4, equal
Point: $\begin{pmatrix} -1 \\ -1 \\ 4 \end{pmatrix}$
Answer: $\begin{pmatrix} \mathbf{-1} \\ \mathbf{-1} \\ \mathbf{4} \end{pmatrix}$

10 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-3x - 14}{(x + 4)(x + 5)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{-3x - 14}{(x + 4)(x + 5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-3x - 14}{(x + 4)(x + 5)} \equiv \frac{A}{x + 4} + \frac{B}{x + 5}$, so $-3x - 14 \equiv A(x + 5) + B(x + 4)$
x = −4: A = −2; x = −5: B = −1
(b) $\int_0^4 \left(\frac{-2}{x + 4} + \frac{-1}{x + 5}\right) dx = [-2\ln(x + 4) - \ln(x + 5)]_0^4$
$= -2\ln\frac{8}{4} - \ln\frac{9}{5} = \ln\left(\frac{5}{36}\right)$

Answer: (a) A = −2, B = −1 (b) $\mathbf{\ln\left(\frac{5}{36}\right)}$