Day 71 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A company employs 50 managers, 130 technicians and 80 operators.
A stratified sample of size 26 is taken. Find the number selected from each group.

Total = 50 + 130 + 80 = 260

Sampling fraction = 26/260 = $\frac{1}{10}$

managers: 50 ÷ 10 = 5
technicians: 130 ÷ 10 = 13
operators: 80 ÷ 10 = 8
Answer: managers 5, technicians 13, operators 8

2 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Expand and simplify $(-1 + 4\sqrt{3})(1 - \sqrt{3})$, giving your answer in the form $a + b\sqrt{3}$.

$(-1 + 4\sqrt{3})(1 - \sqrt{3}) = -1 + \sqrt{3} + 4\sqrt{3} - 4\sqrt{3}\times\sqrt{3}$
$\sqrt{3}\times\sqrt{3} = 3$, so the expression = $-13 + 5\sqrt{3}$
Answer: $-13 + 5\sqrt{3}$

3 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 2 kg is acted on by forces $(3\mathbf{i} + 7\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(2\mathbf{i})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(3\mathbf{i} + 7\mathbf{j}) + \mathbf{P} = 2(2\mathbf{i}) = (4\mathbf{i})$
$\mathbf{P} = \mathbf{i} - 7\mathbf{j}$
$|\mathbf{P}| = \sqrt{(1^2 + -7^2)}$
Answer: $\mathbf{P} = (\mathbf{i} - 7\mathbf{j})$ N; $|\mathbf{P}| = 7.07$ N

4 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A charity raises £100 in week 1. In each following week it raises £10 more than in the previous week.
(a) Find the amount raised in week 11.
(b) Find the total for the first 17 weeks.

(a) Arithmetic sequence with a = 100, d = 10
$u_{11} = 100 + 10 \times 10 = £200$

(b) $S_{17} = \frac{17}{2}(2 \times 100 + 16 \times 10) = £3{,}060$

Answer: (a) £200 (b) £3,060

5 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution u = cos x to find the exact value of

$\int_0^{\frac{\pi}{3}} \sin x \cos^2 x \, dx$.

$\frac{du}{dx} = -\sin x$, so $\sin x \, dx = -du$

Limits: x = 0 gives u = 1, $x = \frac{\pi}{3}$ gives $u = \frac{1}{2}$

$\int_1^{\frac{1}{2}} -u^2 \, du = [-\frac{u^3}{3}]_1^{\frac{1}{2}}$

$= \frac{7}{24}$

Answer: $\frac{7}{24}$

6 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 4)^2 + (y + 1)^2 = 13$.
The point P(6, −4) lies on C.
Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (4, −1). Gradient of radius CP = $\frac{-3}{2} = -\frac{3}{2}$

The tangent is perpendicular to the radius: gradient = $\frac{2}{3}$

$y + 4 = \frac{2}{3}(x - 6)$

Multiply out and collect terms: $2x - 3y - 24 = 0$

Answer: $2x - 3y - 24 = 0$

7 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 2 kg is held on a rough plane ($\tan\alpha = \frac{4}{3}$, $\mu = 0.75$) by a force P N up the plane, as shown.

Take $g = 9.8$ m s^{-2}. The block is about to move up the plane. Find P.

$R = mg\cos\alpha = 19.6 \times 3/5 = 11.76$
Friction (limiting) $F = \mu R = 8.82$
Moving up, friction acts down: $P = mg\sin\alpha + F$
$mg\sin\alpha = 15.68$
Answer: 24.5 N

8 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = xe^{3x}$,
and determine its nature.

$\frac{dy}{dx} = (3x + 1)e^{3x} = 0$

$x = -\frac{1}{3}$

$\frac{d^2y}{dx^2} = 3(3x + 2)e^{3x}$

At $x = -\frac{1}{3}$: $\frac{d^2y}{dx^2} = \frac{3}{e} > 0$, so a minimum

Answer: $\left(-\frac{1}{3}, -\frac{1}{3e}\right)$ minimum

9 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
P(X < 64) = 0.0228 and P(X > 88) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.999$ and $z_2 = 0.9998$
$64 = \mu + (-1.999)\sigma$ and $88 = \mu + 0.9998\sigma$
Subtract: $24 = 2.999\,\sigma$, so $\sigma = 8.00$
$\mu = 64 - (-1.999)(8.003) = 80.0$
Answer: $\mu = 80.0$, $\sigma = 8.00$

10 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = \sqrt{6}\sqrt{x}$, the x-axis
and the lines x = 0 and x = 4.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi\int_0^4 y^2\,dx = \pi\int_0^4 6x\,dx$

$= \pi[3x^2]_0^4$

$= 48\pi$

Answer: 48π