Day 70 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{8x - 19}{(2x - 3)(x - 5)}$ in partial fractions.

$\frac{8x - 19}{(2x - 3)(x - 5)} \equiv \frac{A}{(2x - 3)} + \frac{B}{(x - 5)}$, so $8x - 19 \equiv A(x - 5) + B(2x - 3)$

$x = \frac{3}{2}$: $-7 = A \times (-\frac{7}{2})$, so $A = 2$

$x = 5$: $21 = B \times 7$, so $B = 3$

Answer: $\mathbf{\frac{2}{(2x - 3)} + \frac{3}{(x - 5)}}$

2 Quotient rule (All boards)

Given that $y = x^2e^{-x}$,

find $\frac{dy}{dx}$, and its value when $x = 0$.

Quotient rule with $u = x^2$, $v = e^x$

$u' = 2x$, $v' = e^x$

$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = -x(x - 2)e^{-x}$

When $x = 0$: $\frac{dy}{dx} = 0$

Answer: $\mathbf{\frac{dy}{dx} = -x(x - 2)e^{-x}}$**; value 0**

3 Poisson distribution (CIE)

X ~ Po(6), modelling the number of calls to a helpline per minute.

Find, to 3 significant figures,

(a) $P(X \leq 5)$

(b) $P(X = 5)$

(a) $P(X \leq 5) = P(X = 0) + ... + P(X = 5) = 0.446$

(b) $P(X = 5) = e^{-6} \times \frac{6^5}{5!} = 0.161$

Answer: (a) 0.446 (b) 0.161

4 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{12}{13}$ and $\sin B = \frac{20}{29}$, where A and B are acute,

find the exact value of $\cos(A - B)$.

$\cos A = \frac{5}{13}$ and $\cos B = \frac{21}{29}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$= \frac{5}{13} \times \frac{21}{29} + \frac{12}{13} \times \frac{20}{29} = \frac{345}{377}$

Answer: $\mathbf{\frac{345}{377}}$

5 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 - 6x + 3$ for $x \geq 3$.

Find $f^{-1}(x)$ and state its domain.

Complete the square: $f(x) = (x - 3)^2 - 6$

$y = (x - 3)^2 - 6$, so $(x - 3)^2 = y + 6$

As $x - 3 \geq 0$ take the positive root: $x = 3 + \sqrt{y + 6}$

As $x - 3 \geq 0$, take the positive root: $x = 3 + \sqrt{y + 6}$

Answer: $\mathbf{f^{-1}(x) = 3 + \sqrt{x + 6}}$**, domain** $\mathbf{x \geq -6}$

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6 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 4 kg is held on a rough plane ($\tan\alpha = \frac{3}{4}$, $\mu = 0.5$) by a force P N up the plane, as shown.

Take $g = 9.8$ m s^{-2}. The block is about to move down the plane. Find P.

$R = mg\cos\alpha = 39.2 \times 4/5 = 31.36$
Friction (limiting) $F = \mu R = 15.68$
Moving down, friction acts up: $P + F = mg\sin\alpha$
$mg\sin\alpha = 23.52$
Answer: 7.84 N

7 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of
$(3 + 4x)(3 - 2x)^5$.

$(3 - 2x)^5$ has x^2 coefficient 1080 and x^3 coefficient -720
x^3 term of the product $= 3 \times (-720) + 4 \times 1080$
$= 2160$
Answer: 2160

8 Basic trig equations (degrees) (All boards)

Solve $\cos 2x = 0.3$ for $0 \le x \le 360°$, giving answers to 1 d.p.

Let $\theta = 2x$, so $\cos\theta = 0.3$ with θ in range 0° to 720°
$\theta = 72.54°, 287.46°, 432.54°, 647.46°$
$x = 36.3°, 143.7°, 216.3°, 323.7°$
Answer: x = 36.3°, 143.7°, 216.3°, 323.7°

9 Finding the mean or standard deviation (All boards)

The length X cm of a metal rod is normal with s.d. 1.5.
Given $P(X > 72.1) = 0.025$, find the mean (3 s.f.).

$P(X > 72.1) = 0.025$, so $P(X < 72.1) = 0.975$
$z = -1.960$: $\frac{72.1 - \mu}{1.5} = -1.960$
$\mu = 72.1 + 1.960 \times 1.5 = 75.0$
Answer: μ = 75.0

10 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 50 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 20 to t = 40.
The floors are 875 m apart. Find V, and the deceleration in the final stage.

Area $= \frac{1}{2} \times (50 + 20) \times V = 875$
$70\,V = 1750$ so $V = 25$
Deceleration $= 25 \div 10$
Answer: V = 25; deceleration 2.5 m s^{-2}