Day 7 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Independent and mutually exclusive events (All boards)

A and B are mutually exclusive events with P(A) = 0.4 and P(A ∪ B) = 0.65.
(a) Find P(B).
(b) Find P(A' ∩ B').

(a) Mutually exclusive: P(A ∪ B) = P(A) + P(B)
P(B) = 0.65 − 0.4 = 0.25
(b) P(A' ∩ B') = 1 − P(A ∪ B) = 0.35
Answer: (a) 0.25 (b) 0.35

2 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (2, −4) and (6, 8).
(a) Find the midpoint of AB.
(b) Find the line AB as ax + by + c = 0, with integer a, b, c.

(a) Midpoint = $(\frac{2+6}{2}, \frac{-4+8}{2}) = (4, 2)$

(b) Gradient = $\frac{12}{4} = 3$

y + 4 = 3(x − 2)
Multiply out and collect terms: 3x − y − 10 = 0
Answer: (a) (4, 2) (b) 3x − y − 10 = 0

3 Product rule (All boards)

Given that $y = x^3 \ln x$,
find $\frac{dy}{dx}$, and its value when x = 1.

Product rule with $u = x^3$, $v = \ln x$
$u' = 3x^2$, $v' = \frac{1}{x}$
$\frac{dy}{dx} = u'v + uv' = x^2(3 \ln x + 1)$
When x = 1: $\frac{dy}{dx} = 1$
Answer: $\frac{dy}{dx} = x^2(3 \ln x + 1)$; value 1

4 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 14 pairs of data, $S_{xx} = 200$, $S_{yy} = 150$ and $S_{xy} = -94$.
Find the PMCC (3 s.f.) and describe the correlation.

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{-94}{\sqrt{200 \times 150}}$
$= \frac{-94}{173.21} = -0.543$
r is negative, so there is weak negative correlation
Answer: r = −0.543, weak negative correlation

5 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + px^2 + qx - 6$.
When f(x) is divided by (x + 3) the remainder is −12, and when divided by (x + 1) the remainder is 0. Find p and q.

f(−3) = −12: −54 + 9p − 3q − 6 = −12, so 9p − 3q = 48
f(−1) = 0: −2 + p − q − 6 = 0, so p − q = 8
Solving simultaneously: p = 4, q = −4
Answer: p = 4, q = −4

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6 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 4 kg is pulled across smooth horizontal ice by a rope with tension 39 N, inclined at α above the horizontal, where $\tan\alpha = \frac{5}{12}$.

Take $g = 9.8$ m s^{-2}.

Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos\alpha = 12/13$, $\sin\alpha = 5/13$

(a) Horizontal: $39 \times 12/13 = 4a$

(b) Vertical: $R + 39 \times 5/13 = 4 \times 9.8$

Answer: (a) 9 m s^{-2} (b) 24.2 N

7 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$$\int_0^{\ln 2} \frac{e^x}{e^x + 4}\,dx,$$

giving your answer in the form $a \ln b$.

The numerator is a multiple of the derivative of the denominator

$$\int \frac{e^x}{e^x + 4}\,dx = \ln(e^x + 4)$$

$$= [\ln(e^x + 4)]_0^{\ln 2}$$

$$= \ln\left(\frac{6}{5}\right)$$

Answer: $\ln\left(\frac{6}{5}\right)$

8 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$, a quadratic.

(a) Find the coordinates of the local maximum point of $y = |f(x)|$.

(b) How many solutions has $|f(x)| = 16$?

(a) $y = |f(x)|$ reflects the part of the curve below the x-axis in the x-axis:

$(0, -16)$ becomes $(0, 16)$

(b) The maximum of the middle part is 16. Compare $k = 16$ with 16:

$k = 16$ gives 3 intersections with $y = k$.

Answer: (a) (0, 16) (b) 3

9 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 20 kg rests on a rough horizontal floor, with coefficient of friction 0.6. A horizontal force of 141 N is applied to the box.

Take $g = 9.8$ m s^{-2}.

Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 20 \times 9.8 = 196$, maximum friction $\mu R = 0.6 \times 196 = 117.6$

$141 > 117.6$ so the box moves, $F = 117.6$

$141 - 117.6 = 20a$

Answer: It moves; a = 1.17 m s^{-2}

10 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = x^2e^{1-x} + 3$.

Find the equation of the normal to C at the point where $x = 1$, in the form $y = mx + c$.

$$\frac{dy}{dx} = -x(x - 2)e^{1-x}$$

At $x = 1$: $y = 4$ and $\frac{dy}{dx} = 1$

Normal gradient $= -1 \div 1 = -1$

$y - 4 = -1(x - 1)$, so $y = -x + 5$

Answer: $y = -x + 5$