Day 69 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + 5n + 11$ is odd for all integers n.

$n^2 + 5n = n(n + 5)$
As 5 is odd, n and n + 5 have opposite parity,
so one of them is even and the product n(n + 5) is even.
Even + 11 = odd, so $n^2 + 5n + 11$ is odd.
Answer: n(n + 5) is even, so the expression is odd

2 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 2 : 3, and N lies on OB with ON : NB = 2 : 1.
Find OM and NM in terms of a and b.

AB = b − a, so AM = $\frac{2}{5}$(b − a)

OM = a + AM = $\frac{3a}{5} + \frac{2b}{5}$

ON = $\frac{2b}{3}$, so NM = OM − ON = $\frac{3a}{5} - \frac{4b}{15}$

Answer: OM = $\frac{3a}{5} + \frac{2b}{5}$, NM = $\frac{3a}{5} - \frac{4b}{15}$

3 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A ship moves with constant velocity $(2\mathbf{i} - \mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(-5\mathbf{i} + 5\mathbf{j})$ m.
Find (a) its position vector when t = 2, (b) the value of t when it crosses the line through O in the direction of **i**.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (-5\mathbf{i} + 5\mathbf{j}) + 2(2\mathbf{i} - \mathbf{j}) = (-\mathbf{i} + 3\mathbf{j})$
(b) **j** component zero: 5 − t = 0, so t = 5
Answer: (a) $(-\mathbf{i} + 3\mathbf{j})$ m (b) t = 5 s

4 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 2 and common ratio 4.
Find the smallest value of n for which the sum of the first n terms exceeds 50000.

$S_n = \frac{2(4^n - 1)}{3} > 50000$

$4^n > 75001$

$n > \frac{\lg 75001}{\lg 4} = 8.097$

So the smallest integer n is 9
Answer: n = 9

5 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 4 kg is pulled across smooth horizontal ice by a rope with tension 39 N, inclined at α above the horizontal, where $\tan\alpha = \frac{4}{3}$.

Take g = 9.8 m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

cos α = 3/5, sin α = 4/5
(a) Horizontal: 39 × 3/5 = 4 a
(b) Vertical: R + 39 × 4/5 = 4 × 9.8
Answer: (a) 5.85 m s^{-2} (b) 8 N

6 Conditional probability formula (All boards)

P(B) = 0.25, P(A | B) = 0.25 and P(A) = 0.7.
(a) Find P(A ∩ B).
(b) Find P(B | A).
(c) Find P(A ∪ B).

(a) $P(A \cap B) = P(A \mid B) \times P(B) = 0.25 \times 0.25 = 0.0625$

(b) $P(B \mid A) = \frac{0.0625}{0.7} = \frac{5}{56}$

(c) $P(A \cup B) = 0.7 + 0.25 - 0.0625 = 0.8875$

Answer: (a) 0.0625 (b) $\frac{5}{56}$ (c) 0.8875

7 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(5, −2) and B(0, 3), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:

$25 + (h + 2)^2 = 0 + (h - 3)^2$

Expanding and cancelling h^2: $4h + 29 = -6h + 9$

$h = -2$

$r^2 = CA^2 = 25$

Answer: centre (0, −2), $x^2 + (y + 2)^2 = 25$

8 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(120, 0.5). Use a normal approximation to find P(57 ≤ X ≤ 63), to 3 significant figures.

Y ~ N(np, np(1 − p)) = N(60.00, 30.00), so $\sigma = \sqrt{30.00} = 5.477$
Use a continuity correction.
P(57 ≤ X ≤ 63) ≈ P(56.5 < Y < 63.5) = 0.477
Answer: 0.477

9 Optimisation: open box and tank (All boards)

A closed cylinder has radius r cm and volume 432π cm^3.

Show that its surface area is $S = 2\pi r^2 + \frac{864\pi}{r}$, and find the minimum S.

Height $h = \frac{432}{r^2}$, so $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + \frac{864\pi}{r}$

$\frac{dS}{dr} = 4\pi r - \frac{864\pi}{r^2} = 0$, so $r^3 = 216$ and $r = 6$

$\frac{d^2S}{dr^2} = 4\pi + \frac{1728\pi}{r^3} > 0$, so minimum

$S = 2\pi \times 36 + \frac{864\pi}{6} = 216\pi$ cm^2

Answer: $S = 216\pi$ cm^2

10 Locating roots: change of sign (All boards)

The function f(x) = ln x + x − 6.
Show that f(x) = 0 has a root between x = 4 and x = 4.5.

f(4) = −0.614
f(4.5) = 0.00408
f is continuous and changes sign, so there is a root in the interval
Answer: f(4) < 0 and f(4.5) > 0: sign change