Day 68 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Standard integrals (definite) (All boards)

Find the exact value of

$\int_0^{\ln 2} (3x + 6e^{-2x})\,dx$.

$\int (3x + 6e^{-2x})\,dx = \frac{3x^2}{2} - 3e^{-2x}$

$= [\frac{3x^2}{2} - 3e^{-2x}]_0^{\ln 2}$

$= \frac{3(\ln 2)^2}{2} + \frac{9}{4}$

Answer: $\frac{3(\ln 2)^2}{2} + \frac{9}{4}$

2 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $3x^2 + 24x + 42$ in the form $a(x + p)^2 + q$.

(b) Write down the coordinates of the minimum point of $y = 3x^2 + 24x + 42$.

Take out the factor 3: $3x^2 + 24x + 42 = 3(x^2 + 8x) + 42$

$= 3[(x + 4)^2 - 16] + 42$

$= 3(x + 4)^2 - 6$

$(x + 4)^2 \geq 0$ with equality at $x = -4$, so the minimum is -6 at $x = -4$.

Answer: (a) $3(x + 4)^2 - 6$ (b) $(-4, -6)$

3 Large data set: outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Large data set: mean pressure (hPa) at Camborne has mean 1017, sd 4.

Outliers are more than 2 sd from the mean.

Are the values 1028 and 1007 outliers?

Limits: $1017 \pm 2 \times 4$, so from 1009 to 1025

1028 is outside the limits, so it is an outlier

1007 is outside the limits, so it is an outlier

Answer: 1028: outlier; 1007: outlier

4 Inequalities with exponents and logs (All boards)

£1000 is invested at 4% compound interest per year.

Find the least whole number of years for the investment to exceed £2000.

$1000 \times 1.04^n > 2000$, so $1.04^n > 2$

$n > \frac{\ln 2}{\ln 1.04} = 17.67$

The least whole number is $n = 18$

Answer: 18 years

5 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = -2u_n - 5$, $u_1 = 1$.

(a) Find u_2 and u_3.

(b) Find $\sum_{r=1}^{4} u_r$.

(a) $u_2 = -2 \times 1 - 5 = -7$

$u_3 = -2 \times (-7) - 5 = 9$

(b) $u_4 = -2 \times 9 - 5 = -23$

Sum $= 1 + (-7) + 9 + (-23) = -20$

Answer: (a) $u_2 = -7$, $u_3 = 9$ (b) -20

6 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 5 kg is acted on by forces $(-2\mathbf{i} + \mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(4\mathbf{i} + 4\mathbf{j})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(-2\mathbf{i} + \mathbf{j}) + \mathbf{P} = 5(4\mathbf{i} + 4\mathbf{j}) = (20\mathbf{i} + 20\mathbf{j})$
$\mathbf{P} = 22\mathbf{i} + 19\mathbf{j}$
$|\mathbf{P}| = \sqrt{(22^2 + 19^2)}$
Answer: $\mathbf{P} = (22\mathbf{i} + 19\mathbf{j})$ N; $|\mathbf{P}| = 29.1$ N

7 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = -t - 1$, $y = -t^2 + t$.
The line $y = x - 2$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $-t^2 + t = -t - 3$
$-t^2 + 2t + 3 = 0$, so $-(t - 3)(t + 1) = 0$
$t = -1$ or $t = 3$, giving the points $(0, -2)$ and $(-4, -6)$
Answer: $(0, -2)$ and $(-4, -6)$

8 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sprinter moves in a straight line with constant acceleration. Its initial speed is 11 m s^{-1}, its acceleration is 1.5 m s^{-2} and the motion lasts 12 s.
Find the final speed.

$v = u + at$
$v = 11 + 1.5 \times 12$
Answer: 29 m s^{-1}

9 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass M kg of a substance satisfies $\frac{dM}{dt} = -0.125M$, where t is in years.

Initially M = 80. Find M when t = 10, and the half-life, each to 3 s.f.

$\frac{1}{M}\,dM = -0.125\,dt$, so $\ln M = -0.125t + c$, giving $M = 80e^{-0.125t}$

$t = 10$: $M = 80\,e^{-1.25} = 22.92$

Half-life: $e^{-0.125t} = \frac{1}{2}$, so $t = \frac{\ln 2}{0.125} = 5.545$

Answer: M = 22.9 kg, half-life 5.55 years

10 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
P(X < 36) = 0.1587 and P(X > 46) = 0.0668.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.500$
$36 = \mu + (-0.9998)\sigma$ and $46 = \mu + 1.500\sigma$
Subtract: $10 = 2.500\,\sigma$, so $\sigma = 4.00$
$\mu = 36 - (-0.9998)(4.000) = 40.0$
Answer: $\mu = 40.0$, $\sigma = 4.00$