Day 67 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int \sec^2 3x \, dx$,

(b) the exact value of $\int_0^{\frac{\pi}{12}} \sec^2 3x \, dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{\tan 3x}{3} + c$

(b) $[\frac{\tan 3x}{3}]_0^{\frac{\pi}{12}}$

$= \frac{1}{3}$

Answer: (a) $\frac{\tan 3x}{3} + c$ (b) $\frac{1}{3}$

2 Interpreting regression (AQA, Edexcel, OCR, MEI, Eduqas)

Regression line (x from 2 to 18): $y = 60 - 2.5x$.
x is the temperature, y the gas used per day.
(a) Interpret the gradient.
(b) Estimate y when x = 10 and comment.

(a) The gradient: for each extra °C rise in temperature, y is predicted to decrease by 2.5
(b) $y = 60 + (-2.5) \times 10 = 35$
x = 10 lies within the range of the data (interpolation), so the estimate is reliable
Answer: (a) y decreases by 2.5 per extra °C rise in temperature (b) 35, reliable (interpolation)

3 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{4x - 27}{(2x - 3)(x - 5)}$ in partial fractions.

$\frac{4x - 27}{(2x - 3)(x - 5)} \equiv \frac{A}{(2x - 3)} + \frac{B}{(x - 5)}$, so $4x - 27 \equiv A(x - 5) + B(2x - 3)$

$x = \frac{3}{2}$: $-21 = A \times (-\frac{7}{2})$, so A = 6

x = 5: $-7 = B \times 7$, so B = −1

Answer: $\frac{6}{(2x - 3)} - \frac{1}{(x - 5)}$

4 Solving exponential equations using logarithms (All boards)

Solve $3^x = 16$, giving your answer to 3 s.f.

Take logs: $x \ln 3 = \ln 16$

$x = \frac{\ln 16}{\ln 3} = 2.524$

Answer: x = 2.52

5 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x - 6| = |x + 4|$.

Either $2x - 6 = x + 4$ or $2x - 6 = -(x + 4)$
$2x - 6 = x + 4$ gives x = 10
$2x - 6 = -(x + 4)$ gives $x = \frac{2}{3}$

Answer: x = $\frac{2}{3}$ and x = 10

6 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 2 kg is pulled across smooth horizontal ice by a rope with tension 15 N, inclined at α above the horizontal, where $\tan\alpha = \frac{3}{4}$.

Take $g = 9.8$ m s^{-2}.

Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos\alpha = 4/5$, $\sin\alpha = 3/5$

(a) Horizontal: $15 \times 4/5 = 2a$

(b) Vertical: $R + 15 \times 3/5 = 2 \times 9.8$

Answer: (a) 6 m s^{-2} (b) 10.6 N

7 Binomial: smallest n (All boards)

Given that each component fails independently with probability 0.1, find the least number of components needed for the probability of at least one success to exceed 0.95.

$P(\text{at least one}) = 1 - 0.9^n > 0.95$

$0.9^n < 0.05$

$n > \frac{\log 0.05}{\log 0.9} = 28.43$

Answer: n = 29

8 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The equation $x^2 + kx + 9 = 0$ has equal roots.

Find the possible values of k.

Equal roots: $b^2 - 4ac = 0$

$k^2 - 4 \times 1 \times 9 = 0$

$k^2 = 36$

$k = \pm 6$

Answer: k = ±6

9 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\tan^2 x - \sec x - 4 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $2\sec^2 x - \sec x - 6 = 0$

$(\sec x - 2)(2\sec x + 3) = 0$, so $\sec x = 2$ or $-\frac{3}{2}$

$\sec x = 2$, so $\cos x = \frac{1}{2}$: $x = 60.0°, 300.0°$

$\sec x = -\frac{3}{2}$, so $\cos x = -\frac{2}{3}$: $x = 131.8°, 228.2°$

Answer: x = 60.0°, 131.8°, 228.2°, 300.0°

10 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 4 m and mass 50 kg rests horizontally on supports at C and D, where AC = 0.5 m and DB = 1.5 m.

A girl of mass 20 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero

Moments about D: $50\,g \times (2.5 - 2) = 20\,g \times x$

$50 \times 0.5 = 20x$

Answer: 1.25 m