**Day 66** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Free fall from rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is dropped from rest from a window, 19.6 m above the ground.
Take g = 9.8 m $s^{-2}$.
Find (a) the time taken to reach the ground, (b) its speed on landing.

(a) $s = ut + \frac{1}{2}at^2$: $19.6 = \frac{1}{2} \times 9.8 \times t^2$
$t^2 = 19.6 \div 4.9 = 4$, so $t = 2$
(b) $v = u + at = 9.8 \times 2$
**Answer: (a) 2 s (b) 19.6 m $s^{-1}$**

## 2 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (−6, −2), (−2, −1) and (−3, 3).
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB = $\frac{-1}{-4} = \frac{1}{4}$

Gradient BC = $\frac{4}{-1} = -4$

$\frac{1}{4} \times (-4) = -1$, so AB and BC are perpendicular.

(b) AB = $\sqrt{17}$ and BC = $\sqrt{17}$
Area = $\frac{1}{2} \times AB \times BC = \frac{17}{2}$

**Answer: (b) area = $\frac{17}{2}$**

## 3 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = \frac{2}{3}$ and θ is acute,
find the exact value of sec θ.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
$\tan\theta = \frac{2}{3}$, so $\sec^2\theta = 1 + \frac{4}{9} = \frac{13}{9}$

$\sec\theta = \frac{\sqrt{13}}{3}$ (positive as θ is acute)

**Answer: $\frac{\sqrt{13}}{3}$**

## 4 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(\mathbf{i} - 4\mathbf{j})$ N and $(-4\mathbf{i} + 8\mathbf{j})$ N act on a particle of mass 2 kg.
Find the magnitude of the acceleration of the particle.

Resultant = $(\mathbf{i} - 4\mathbf{j}) + (-4\mathbf{i} + 8\mathbf{j}) = -3\mathbf{i} + 4\mathbf{j}$
$|R| = \sqrt{(-3^2 + 4^2)} = 5$ N
$a = F/m = 5 \div 2$
**Answer: 2.5 m $s^{-2}$**

## 5 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + px^2 + qx + 6$.
When f(x) is divided by (x + 3) the remainder is −54, and when divided by (x − 1) the remainder is 2. Find p and q.

$f(-3) = -54$: $-54 + 9p - 3q + 6 = -54$, so $9p - 3q = -6$
$f(1) = 2$: $2 + p + q + 6 = 2$, so $p + q = -6$
Solving simultaneously: $p = -2$, $q = -4$
**Answer: $p = -2$, $q = -4$**

## 6 Double angle equations (All boards)

Solve $\cos 2x = \sin x$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $1 - 2\sin^2 x - \sin x = 0$
$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$: $x = 30.0°, 150.0°$

$\sin x = -1$: $x = 270.0°$

**Answer: $x = 30.0°, 150.0°, 270.0°$**

## 7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 5 g. Claimed mean 200 g; 16 packets have mean 201.9 g.
Test at the 5% level whether the true mean is greater than 200.

$H_0: \mu = 200$, $H_1: \mu > 200$. Under $H_0$, $\overline{X} \sim N(200, \frac{5^2}{16})$

$z = \frac{201.9 - 200}{5/\sqrt{16}} = 1.520$

$P(Z > 1.520) = 0.0643 > 0.05$

Do not reject $H_0$: there is insufficient evidence that the mean has increased

**Answer: do not reject $H_0$: insufficient evidence of an increase in the mean**

## 8 Intersection of two lines in 3D (CIE)

Line $l_1$: $\mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$.

Line $l_2$: $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$.

Show that $l_1$ and $l_2$ intersect and find the point.

i: $-3 + 3\lambda = 0 + 3\mu$
j: $1 + 0\lambda = 0 + (-1)\mu$
Solving i and j: $\lambda = 0$, $\mu = -1$
k check: $3 + 3(0) = 3$ and $3 + 0(-1) = 3$, equal

Point: $\begin{pmatrix} -3 \\ 1 \\ 3 \end{pmatrix}$

**Answer:** $\begin{pmatrix} \mathbf{-3} \\ \mathbf{1} \\ \mathbf{3} \end{pmatrix}$

## 9 Poisson: changing the interval (CIE)

Customers arrive at a shop at random at an average rate of 4 per hour.
Find the probability that in 30 minutes there are exactly 2 (3 s.f.).

Rate in 30 minutes: $\lambda = 4 \times \frac{30}{60} = 2$

$X \sim \text{Po}(2)$

$P(X = 2) = e^{-2} \times \frac{2^2}{2!} = 0.271$

**Answer: 0.271**

## 10 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 3t + 1$, $y = t^2 + t - 3$.
The line $y = 2x - 9$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $t^2 + t - 3 = 6t - 7$
$t^2 - 5t + 4 = 0$, so $(t - 4)(t - 1) = 0$
$t = 1$ or $t = 4$, giving the points $(4, -1)$ and $(13, 17)$
**Answer: (4, −1) and (13, 17)**