**Day 65** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Horizontal projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown horizontally at 30 m $s^{-1}$ from the top of a building 30.625 m above level ground.
Take g = 9.8 m $s^{-2}$.
Find the time of flight and the horizontal distance from the base where it lands.

Vertical: $30.625 = \frac{1}{2} \times 9.8 \times t^2$, $t^2 = 6.25$
t = 2.5 s
Horizontal: x = 30 × 2.5
**Answer: 2.5 s, 75 m**

## 2 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(4x)^3 \times 6x^3}{4x^2}$.

$(4x)^3 = 64x^3$
Numerator = $64 \times 6x^6 = 384x^6$
Divide: $\frac{384}{4} = 96$ and $x^6 \div x^2 = x^4$
**Answer: $96x^4$**

## 3 Normal distribution probabilities (All boards)

The mass X g of a flour bag is normal with mean 500, s.d. 3.
Find, to 3 s.f., the probability a value is
(a) less than 506,
(b) between 506 and 509.

$X \sim N(500, 3^2)$
(a) P(X < 506) = 0.977
(b) P(506 < X < 509) = 0.9987 − 0.9772 = 0.0214
**Answer: (a) 0.977 (b) 0.0214**

## 4 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 3 kg is pulled along rough horizontal ground by a horizontal force of 22.2 N. It accelerates at 2.5 m $s^{-2}$.
Take g = 9.8 m $s^{-2}$.
Find the coefficient of friction between the crate and the ground.

R = mg = 3 × 9.8 = 29.4
Resultant: 22.2 − F = 3 × 2.5, so F = 14.7
F = μR: μ = 14.7 ÷ 29.4
**Answer: 0.5**

## 5 Conditional probability formula (All boards)

P(B) = 0.3, P(A | B) = 0.6 and P(A) = 0.5.
(a) Find P(A ∩ B).
(b) Find P(B | A).
(c) Find P(A ∪ B).

(a) P(A ∩ B) = P(A | B) × P(B) = 0.6 × 0.3 = 0.18
(b) $P(B \mid A) = \frac{0.18}{0.5} = 0.36$
(c) P(A ∪ B) = 0.5 + 0.3 − 0.18 = 0.62
**Answer: (a) 0.18 (b) 0.36 (c) 0.62**

## 6 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|3x + 1| = |x + 1|$.

Either $3x + 1 = x + 1$ or $3x + 1 = -(x + 1)$

$3x + 1 = x + 1$ gives $x = 0$

$3x + 1 = -(x + 1)$ gives $x = -\frac{1}{2}$

**Answer: $x = -\frac{1}{2}$ and $x = 0$**

## 7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $2\cos\theta + 3\sin\theta \equiv \sqrt{13}\cos(\theta - 56.3°)$,
solve $2\cos\theta + 3\sin\theta = -2.5$ for $0 \le \theta \le 360°$. (1 d.p.)

$\sqrt{13}\cos(\theta - 56.3°) = -2.5$, so $\cos(\theta - 56.3°) = \frac{-2.5}{\sqrt{13}}$

$\theta - 56.3° = \pm 133.9°$

$\theta = 56.3° + 133.9° = 190.2°$

$\theta = 56.3° - 133.9° = -77.6°$, so $\theta = 282.4°$

**Answer: $\theta$ = 190.2°, 282.4°**

## 8 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A new car costs £25000. Its value falls by 10% each year.
(a) Find its value after 4 years (nearest £).
(b) In which year does it first fall below £3000?

(a) Value = $25000 \times 0.9^4 = £16{,}403$

(b) $25000 \times 0.9^n < 3000$, so $n \lg 0.9 < \lg(3000 \div 25000)$

$n > \lg(3000 \div 25000) \div \lg 0.9 = 20.124$ (dividing by a negative number)

So $n = 21$

**Answer: (a) £16,403 (b) 21 years**

## 9 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small $\theta$ (in radians), use $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{\theta^2}{2}$

to find the value of $\lim_{\theta \to 0} \frac{1 - \cos 6\theta}{\theta \sin\theta}$.

$1 - \cos 6\theta \approx 1 - (1 - \frac{6^2\theta^2}{2}) = \frac{36\theta^2}{2}$

$\theta \sin\theta \approx \theta \times \theta = \theta^2$

Ratio $= \frac{36\theta^2}{(2 \times 1\theta^2)} = 18$

**Answer: 18**

## 10 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = e^{2x} - 8e^x$,
and determine its nature.

$\frac{dy}{dx} = 2(e^x - 4)e^x = 0$

$x = \ln 4$

$\frac{d^2y}{dx^2} = 4(e^x - 2)e^x$

At $x = \ln 4$: $\frac{d^2y}{dx^2} = 32 > 0$, so a minimum

**Answer: (ln 4, −16) minimum**