Day 64 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic series is 11 and the common difference is 7.
The sum of the first n terms is 900. Find the value of n.

$S_n = \frac{n}{2}(2 \times 11 + (n - 1) \times 7) = 900$

$7n^2 + 15n - 1800 = 0$
$(n - 15)(7n + 120) = 0$
n must be positive, so n = 15
Answer: n = 15

2 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.
(a) Describe the correlation.
(b) Why should y = 12 − x not be used for x = 20?

(a) The points lie loosely around a straight line falling: weak negative correlation
(b) The data cover x = 1 to 10; x = 20 is outside this range
so using the line would be extrapolation and may be unreliable
Answer: (a) weak negative correlation (b) extrapolation outside the range of the data

3 Two particles over a pulley (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 5 kg and 9 kg hang on a light string over a smooth fixed pulley, as shown, and are released from rest.
Take g = 9.8 m s^{-2}.
Find the acceleration and the tension in the string.

Heavier: 9 g − T = 9 a
Lighter: T − 5 g = 5 a
Add: 4 g = 14 a, so a = 2.8
T = 5(9.8 + 2.8)
Answer: a = 2.8 m s^{-2}; T = 63 N

4 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 10 pairs of values, $\bar{x} = 12$, $\bar{y} = 40$, $S_{xx} = 20$ and $S_{xy} = -30$.
(a) Find the regression line of y on x in the form y = a + bx.
(b) Estimate y when x = 14.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{-30}{20} = -1.5$

$a = \bar{y} - b\bar{x} = 40 - -1.5 \times 12 = 58$
$y = 58 - 1.5x$
(b) $y = 58 + (-1.5) \times 14 = 37$
Answer: y = 58 − 1.5x; y = 37

5 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point (1, −2) and B is the point (−3, 5).
Find the perpendicular bisector of AB as ax + by + c = 0, with integer a, b, c.

Midpoint of AB = $(-1, \frac{3}{2})$

Gradient of AB = $\frac{7}{-4} = -\frac{7}{4}$

Perpendicular gradient = $\frac{4}{7}$

$y - \frac{3}{2} = \frac{4}{7}(x + 1)$, which gives $8x - 14y + 29 = 0$

Answer: 8x − 14y + 29 = 0

6 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = e^{2x} - 2e^{x}$, and determine its nature.

$\frac{dy}{dx} = 2(e^{x} - 1)e^{x} = 0$

$x = 0$

$\frac{d^2y}{dx^2} = 2(2e^{x} - 1)e^{x}$

At $x = 0$: $\frac{d^2y}{dx^2} = 2 > 0$, so a minimum

Answer: $(0, -1)$ minimum

7 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which

$4x \leq -2$ and $x^2 - 2x - 35 \geq 0$.

$4x \leq -2$ gives $x \leq -\frac{1}{2}$

$x^2 - 2x - 35 \geq 0$: $(x + 5)(x - 7) \geq 0$ gives $x \leq -5$ or $x \geq 7$

Both must hold: $x \leq -5$

Answer: $x \leq -5$

8 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 3 kg lies on a rough horizontal table (coefficient of friction 0.4). It is joined by a light string over a smooth pulley to a particle of mass 3 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.4 \times 3g = 3a$

Hanging: $3g - T = 3a$

Add: $3g - 1.2g = 6a$

$a = 2.94$

$T = 3(9.8 - 2.94)$

Answer: $a = 2.94$ m s^{-2}; $T = 20.58$ N

9 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A chord AB subtends an angle of 1.9 radians at the centre O of a circle of radius 8 cm.

Find the perimeter of the minor segment, to 3 s.f.

Arc $= r\theta = 8 \times 1.9 = 15.2$

Chord AB $= 2r \sin(\frac{\theta}{2}) = 13.01$

Perimeter $= 15.2 + 13.01 = 28.21$ cm

Answer: 28.2 cm

10 Optimisation: open box and tank (All boards)

A closed cylinder has radius r cm and volume 250π cm^3.

Show that its surface area is $S = 2\pi r^2 + \frac{500\pi}{r}$, and find the minimum S.

Height $h = \frac{250}{r^2}$, so $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + \frac{500\pi}{r}$

$\frac{dS}{dr} = 4\pi r - \frac{500\pi}{r^2} = 0$, so $r^3 = 125$ and $r = 5$

$\frac{d^2S}{dr^2} = 4\pi + \frac{1000\pi}{r^3} > 0$, so minimum

$S = 2\pi \times 25 + \frac{500\pi}{5} = 150\pi$ cm^2

Answer: $S = 150\pi$ cm^2