Day 63 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Work-energy: sliding to rest (CIE)

A block of mass 5 kg is projected along a rough horizontal surface with speed 2.8 m s^{-1} and comes to rest after sliding 4 m.

Take g = 9.8 m s^{-2}.

Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE

$\mu \times 5 \times 9.8 \times 4 = \frac{1}{2} \times 5 \times 2.8^2$

$196\mu = 19.6$

Answer: 0.1

2 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Values of y = f(x) are shown.

x: 1, 2, 3, 4, 5, 6

y: 2.9, 2.5, 2.1, 1.1, 2.7, 3

Estimate $\int_1^6 f(x)\,dx$ using the trapezium rule.

h = 1

Area $\approx \frac{h}{2}[y_0 + y_5 + 2(y_1 + \ldots + y_4)]$

$= \frac{1}{2}[2.9 + 3 + 2(2.5 + 2.1 + 1.1 + 2.7)]$

= 11.35

Answer: 11.35

3 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 5 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{5}{13}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 5$, so $P \times \frac{5}{13} = 5$

P = 13

Resolve vertically: $Q = P\sin\theta = 13 \times \frac{12}{13}$

Answer: P = 13 N; Q = 12 N

4 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that if n is odd then $n^2 + 9n + 5$ is odd.

Let n = 2k + 1, where k is an integer.

$n^2 = 4k^2 + 4k + 1$

$9n = 18k + 9$

Total = $4k^2 + 22k + 15$

4k + 22 is even and 15 is odd, so the total is odd.

Answer: the expression is odd

5 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = 2x + \ln(2x^2 - 1)$.

Find the equation of the tangent to C at the point where x = 1, in the form y = mx + c.

$\frac{dy}{dx} = 2\left(\frac{2x}{2x^2 - 1} + 1\right)$

At x = 1: y = 2 and $\frac{dy}{dx} = 6$

$y - 2 = 6(x - 1)$, so $y = 6x - 4$

Answer: $y = 6x - 4$

6 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $7x - 2y = -7$ and line l_2 has equation $7x + 6y = 49$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (1, 7)
l_1 meets y = 0 at x = −1, and l_2 meets y = 0 at x = 7
Base = 8, height = 7
Area = $\frac{1}{2}$ × base × height = 28
Answer: 28

7 Double angle equations (All boards)

Solve sin 2x = cos x for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: 2 sin x cos x − cos x = 0
cos x(2 sin x − 1) = 0
cos x = 0: x = 90.0°, 270.0°
sin x = $\frac{1}{2}$: x = 30.0°, 150.0°
Answer: x = 30.0°, 90.0°, 150.0°, 270.0°

8 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{2x^2 - 4x - 18}{(x + 3)(x - 3)}$ in the form $P + \frac{Q}{(x + 3)} + \frac{R}{(x - 3)}$.

$2x^2 - 4x - 18 = 2(x + 3)(x - 3) + (-4x)$, so P = 2
$\frac{2x^2 - 4x - 18}{(x + 3)(x - 3)} \equiv 2 + \frac{Q}{(x + 3)} + \frac{R}{(x - 3)}$
x = −3 gives Q = −2; x = 3 gives R = −2
Answer: $2 + -\frac{2}{(x + 3)} - \frac{2}{(x - 3)}$

9 Poisson: changing the interval (CIE)

Meteors are seen at random at an average rate of 1.5 per hour.
Find the probability that in 30 minutes there are at least 3 (3 s.f.).

Rate in 30 minutes: $\lambda = 1.5 \times \frac{30}{60} = 0.75$
X ~ Po(0.75)
$P(X \geq 3) = 1 - P(X \leq 2) = 0.0405$
Answer: 0.0405

10 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
P(X < 35) = 0.1587 and P(X > 45) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$
$35 = \mu + (-0.9998)\sigma$ and $45 = \mu + 0.9998\sigma$
Subtract: $10 = 2.000\,\sigma$, so $\sigma = 5.00$
$\mu = 35 - (-0.9998)(5.001) = 40.0$
Answer: $\mu = 40.0$, $\sigma = 5.00$