Day 62 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{3x^2}{8}$ for $0 \le x \le 2$, and 0 otherwise.

(a) Find $P(X > 1)$.
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^3}{8}$

(a) $P(X > 1) = 1 - F(1) = \frac{7}{8}$

(b) $F(m) = 0.5$: $\frac{m^3}{8} = \frac{1}{2}$

$m = 1.59$

Answer: (a) $\frac{7}{8}$ (b) 1.59

2 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A van of mass 2000 kg moves along a straight horizontal road. The engine produces a driving force of 2400 N and the resistance to motion is 600 N.
Find the acceleration of the van.

Resultant force = 2400 − 600 = 1800 N
F = ma: 1800 = 2000 a

Answer: 0.9 m s^{-2}

3 Transformations of a turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

y = f(x) has a maximum point at (6, −4).
Find the turning point on the curve:
(a) y = f(3x) (b) y = f(x) − 3
(c) y = f(−x)

(a) y = f(3x): the point (6, −4) maps to (2, −4)
(b) y = f(x) − 3: the point (6, −4) maps to (6, −7)
(c) y = f(−x): the point (6, −4) maps to (−6, −4)

Answer: (a) (2, −4) (b) (6, −7) (c) (−6, −4)

4 Poisson: changing the interval (CIE)

Customers arrive at a shop at random at an average rate of 2.5 per hour.
Find the probability that in 45 minutes there are exactly 3 (3 s.f.).

Rate in 45 minutes: $\lambda = 2.5 \times \frac{45}{60} = 1.875$

$X \sim Po(1.875)$

$P(X = 3) = e^{-1.875} \times \frac{1.875^3}{3!} = 0.168$

Answer: 0.168

5 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Expand and simplify $(2 + 3\sqrt{2})(-1 + 4\sqrt{2})$, giving your answer in the form $a + b\sqrt{2}$.

$(2 + 3\sqrt{2})(-1 + 4\sqrt{2}) = -2 + 8\sqrt{2} - 3\sqrt{2} + 12\sqrt{2}\times\sqrt{2}$
$\sqrt{2}\times\sqrt{2} = 2$, so the expression $= 22 + 5\sqrt{2}$

Answer: $22 + 5\sqrt{2}$

6 Exponential quadratics and ln equations (All boards)

Solve $\ln(5x + 2) = 2$, giving your answer to 3 s.f.

$5x + 2 = e^2$

$x = \frac{e^2 - 2}{5} = 1.078$

Answer: x = 1.08

7 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A new car costs £20000. Its value falls by 15% each year.
(a) Find its value after 3 years (nearest £).
(b) In which year does it first fall below £8000?

(a) Value = $20000 \times 0.85^3 = £12{,}283$

(b) $20000 \times 0.85^n < 8000$, so $n \lg 0.85 < \lg(8000 \div 20000)$
$n > \lg(8000 \div 20000) \div \lg 0.85 = 5.638$ (dividing by a negative number)
So $n = 6$

Answer: (a) £12,283 (b) 6 years

8 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (2 kg) and B (6 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 5 m above the ground, then released.
Take $g = 9.8$ m s^{-2}.
Find the speed with which B hits the ground.

$a = (6 - 2)g \div (6 + 2) = 4.9$
$v^2 = 0 + 2 \times 4.9 \times 5 = 49$
$v = 7$

Answer: 7 m s^{-1}

9 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = Ae^{kx}$. A graph of ln y against x is a straight line through (0, 1.5) and (4, 3.1).
Find A to 3 s.f. and k.

$\ln y = \ln A + kx$, so $\ln A = 1.5$ and $A = e^{1.5} = 4.482$

k = gradient = $\frac{3.1 - 1.5}{4 - 0} = 0.4$

Answer: A = 4.48, k = 0.4

10 Locating roots: change of sign (All boards)

The function $f(x) = x^3 - 1x - 8$.
Show that $f(x) = 0$ has a root between $x = 2$ and $x = 2.5$.

$f(2) = -2.00$
$f(2.5) = 5.12$
f is continuous and changes sign, so there is a root in the interval

Answer: f(2) < 0 and f(2.5) > 0: sign change