Day 61 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chain rule (All boards)

Given that $y = \frac{1}{(3x + 1)}$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = -1$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = -\frac{3}{(3x + 1)^2}$

When $x = -1$: gradient $= -\frac{3}{4}$

Answer: $\frac{dy}{dx} = -\frac{3}{(3x + 1)^2}$; **gradient** $-\frac{3}{4}$

2 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$. A and B are the turning points. Find the turning points of:

(a) $y = f(x) + 5$ (b) $y = f(\frac{x}{2})$

(a) $y = f(x) + 5$: A(1, 4) maps to (1, 9), B(5, −5) maps to (5, 0)

(b) $y = f(\frac{x}{2})$: A(1, 4) maps to (2, 4), B(5, −5) maps to (10, −5)

Answer: (a) (1, 9), (5, 0) (b) (2, 4), (10, −5)

3 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.
(a) Describe the correlation.
(b) Why should $y = 2 + x$ not be used for $x = 15$?

(a) The points lie loosely around a straight line rising: weak positive correlation
(b) The data cover $x = 1$ to 10; $x = 15$ is outside this range
so using the line would be extrapolation and may be unreliable
Answer: (a) weak positive correlation (b) extrapolation outside the range of the data

4 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = 2x^2 + e^{2x} + 1$.
Find the equation of the tangent to C at the point where $x = 0$, in the form $y = mx + c$.

$\frac{dy}{dx} = 4x + 2e^{2x}$

At $x = 0$: $y = 2$ and $\frac{dy}{dx} = 2$

$y - 2 = 2x$, so $y = 2x + 2$

Answer: y = 2x + 2

5 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 - 5x + 6$ and the line $y = -4x + 6$ meet at the points A and B.
Find the coordinates of A and B, and the exact length AB.

$x^2 - 5x + 6 = -4x + 6$
$(x - 1)x = 0$
$x = 0$ or $x = 1$
$y = -4x + 6$: $y = 6$ and $y = 2$
$AB^2 = 1^2 + -4^2 = 17$

Answer: A(0, 6), B(1, 2), AB $= \sqrt{17}$

6 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 4 m and mass 30 kg is hinged at A and held horizontal by a light string attached at B.

The string makes an angle θ with the rod, where $\sin\theta = \frac{3}{5}$. Take $g = 9.8$ m s^{-2}. Find the tension in the string.

Moments about A: $T \sin\theta \times 4 = mg \times 2$

$T \times \frac{3}{5} \times 4 = 294 \times 2$

$T = 588 \div 2.4$

Answer: 245 N

7 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $3x^2y + 2xy + 2y^3 = -78$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at $(-2, -3)$.

Differentiate with respect to x: $3x^2\frac{dy}{dx} + 6xy + 2x\frac{dy}{dx} + 6y^2\frac{dy}{dx} + 2y = 0$

$\frac{dy}{dx} = \frac{2y(-3x - 1)}{3x^2 + 2x + 6y^2}$

At $(-2, -3)$: gradient $= -\frac{15}{31}$

Answer: $\frac{dy}{dx} = \frac{2y(-3x - 1)}{3x^2 + 2x + 6y^2}$; gradient $-\frac{15}{31}$

8 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 12 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{12}{13}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 12$, so $P \times \frac{12}{13} = 12$

$P = 13$

Resolve vertically: $Q = P\sin\theta = 13 \times \frac{5}{13}$

Answer: P = 13 N; Q = 5 N

9 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $4x + 3y = k$ is a tangent to the circle

$(x - 1)^2 + (y + 3)^2 = 25$.

Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (4, 3).
A unit step in this direction is (4, 3)/5, so the points of contact are
$(1, -3) \pm 5 \times (4, 3)/5 = (1, -3) \pm (4, 3)$
T = (5, 0) or $(-3, -6)$, and $k = 4x + 3y$ gives:
$k = 20$ or $k = -30$

Answer: k = 20 or k = −30

10 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
$P(X < 80) = 0.0228$ and $P(X > 110) = 0.1587$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.999$ and $z_2 = 0.9998$
$80 = \mu + (-1.999)\sigma$ and $110 = \mu + 0.9998\sigma$
Subtract: $30 = 2.999\,\sigma$, so $\sigma = 10.0$
$\mu = 80 - (-1.999)(10.00) = 100$

Answer: μ = 100, σ = 10.0