Day 60 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the ages, a years, of 9 members of a club gives $\Sigma a = 221$ and $\Sigma a^2 = 6723$.
Find the mean and the standard deviation (3 s.f.).

Mean = $221 \div 9 = 24.6$

Variance = $\frac{6723}{9} - (24.556)^2 = 144.0$

Standard deviation = $\sqrt{144.0} = 12.0$

Answer: mean 24.6, standard deviation 12.0

2 Double angle equations (All boards)

Solve $\cos 2x = \cos x$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $2\cos^2 x - 1 - \cos x = 0$
$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$: $x = 120.0°, 240.0°$

$\cos x = 1$: $x = 0.0°, 360.0°$

Answer: $x = 0.0°, 120.0°, 240.0°, 360.0°$

3 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 + 10x + 24 > 0$.

Factorise: $(x + 4)(x + 6) > 0$
Critical values: $x = -6$ and $x = -4$
Sketch the ∪-shaped parabola: $x < -6$ or $x > -4$
Answer: $x < -6$ or $x > -4$

4 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{5}{13}$ and $\cos B = \frac{4}{5}$, where A and B are acute,
find the exact value of $\sin(A - B)$.

$\cos A = \frac{12}{13}$ and $\sin B = \frac{3}{5}$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$
$= \frac{5}{13} \times \frac{4}{5} - \frac{12}{13} \times \frac{3}{5} = -\frac{16}{65}$

Answer: $-\frac{16}{65}$

5 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = t$, $y = \frac{4}{t}$, $t \neq 0$.

Find a Cartesian equation of the curve.

Multiply the two equations: $xy = t \times \frac{4}{t} = 4$

So $y = \frac{4}{x}$

Answer: $y = \frac{4}{x}$

6 Instantaneous rest and displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line from O. At time t s its velocity is $6t^2 - 54t + 120$ m s^{-1}.
Find the first time P is at instantaneous rest, and its distance from O then.

$v = 0$: $6(t - 4)(t - 5) = 0$, so $t = 4$ or $t = 5$
First time $t = 4$

$s = \int_0^4 v\,dt = [\,2t^3 - 27t^2 + 120t\,]$ from 0 to 4

Answer: t = 4 s; 176 m

7 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of
$(3 + 3x)(2 + x)^6$.

$(2 + x)^6$ has x^2 coefficient 240 and x^3 coefficient 160
x^3 term of the product = 3 × 160 + 3 × 240
= 1200
Answer: 1200

8 Jumping to a different journey: two-stage motion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train accelerates uniformly from rest at 0.5 m s^{-2} for 6 s, then travels at constant speed for 15 s, and finally decelerates uniformly at 0.5 m s^{-2} to rest.
Find the total distance travelled and the total time of the journey.

Max speed = 0.5 × 6 = 3
Stage 1: ½ × 6 × 3 = 9; stage 2: 3 × 15 = 45
Stage 3: time = 3 ÷ 0.5 = 6, distance = ½ × 3 × 6 = 9
Answer: 63 m; 27 s

9 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 14 pairs of data, $S_{xx} = 90$, $S_{yy} = 200$ and $S_{xy} = 73$.
Find the PMCC (3 s.f.) and describe the correlation.

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{73}{\sqrt{90 \times 200}}$$

$$= \frac{73}{134.16} = 0.544$$

r is positive, so there is weak positive correlation
Answer: r = 0.544, weak positive correlation

10 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\tan^2 x + \sec x - 13 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $2\sec^2 x + \sec x - 15 = 0$

$(\sec x + 3)(2\sec x - 5) = 0$, so $\sec x = -3$ or $\frac{5}{2}$

$\sec x = -3$, so $\cos x = -\frac{1}{3}$: x = 109.5°, 250.5°

$\sec x = \frac{5}{2}$, so $\cos x = \frac{2}{5}$: x = 66.4°, 293.6°

Answer: x = 66.4°, 109.5°, 250.5°, 293.6°