Day 6 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
P(X < 32) = 0.1587 and P(X > 56) = 0.0228.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.999$
$32 = \mu + (-0.9998)\sigma$ and $56 = \mu + 1.999\sigma$
Subtract: $24 = 2.999\,\sigma$, so $\sigma = 8.00$
$\mu = 32 - (-0.9998)(8.003) = 40.0$
Answer: $\mu = 40.0$, $\sigma = 8.00$

2 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^4 (4x^3 + 1 - \frac{2}{x^2})\,dx$.

$\int (4x^3 + 1 - \frac{2}{x^2})\,dx = x^4 + x + \frac{2}{x}$

$= [x^4 + x + \frac{2}{x}]_1^4$

$= \frac{513}{2}$

Answer: $\frac{513}{2}$

3 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:
(a) $y = f(-x)$
(b) $y = f(\frac{x}{2})$
(c) $y = f(x + 5)$

(a) $y = f(-x)$ is a reflection in the y-axis.

(b) $y = f(\frac{x}{2})$ is a stretch, scale factor 2, parallel to the x-axis.

(c) $y = f(x + 5)$ is a translation $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$.

Answer: (a) reflection in the y-axis (b) stretch, scale factor 2, parallel to the x-axis (c) translation $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$

4 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = ax^n$. A graph of ln y against ln x is a straight line with gradient 4 that passes through (0, ln 4).
Find the values of a and n.

$\ln y = \ln a + n \ln x$
gradient = n = 4
intercept $\ln a = \ln 4$, so a = 4
Answer: a = 4, n = 4

5 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (2t^2 + 3)\mathbf{i} + (6t - 24)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0
$6t - 24 = 0$, so t = 4
$\mathbf{v} = (2 \times 4^2 + 3)\mathbf{i} = 35\mathbf{i}$
Answer: t = 4 s, speed 35 m s^{-1}

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6 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x^2 - 2 \geq 0$.

Factorise: $2(x - 1)(x + 1) \geq 0$
Critical values: $x = -1$ and $x = 1$
Sketch the ∪-shaped parabola: $x \leq -1$ or $x \geq 1$
Answer: $x \leq -1$ or $x \geq 1$

7 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ax^n$. A plot of $\log_{10} y$ against $\log_{10} x$ is a straight line with intercept 0.778 and gradient 1.5.
Find a (2 s.f.) and n, then estimate y when $x = 16$ (3 s.f.).

$\log_{10} y = \log_{10} a + n \log_{10} x$

n = gradient = 1.5; $\log_{10} a = 0.778$, so $a = 10^{0.778} = 6.0$

$y = 5.998 \times 16^{1.5} = 384$
Answer: a = 6.0, n = 1.5, y = 384

8 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 4.9 m s^{-1} from a point 29.4 m above horizontal ground.
Take g = 9.8 m s^{-2}.
Find the time before it hits the ground.

Taking up as positive: $-29.4 = 4.9t - 4.9t^2$
$4.9t^2 - 4.9t - 29.4 = 0$
$t = (4.9 \pm \sqrt{600.25}) \div 9.8 = (4.9 \pm 24.5) \div 9.8$
Positive root only.
Answer: 3 s

9 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = -t$, $y = t^2 - t + 4$.
The line $y = -2x + 2$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $t^2 - t + 4 = 2t + 2$
$t^2 - 3t + 2 = 0$, so $(t - 2)(t - 1) = 0$
$t = 1$ or $t = 2$, giving the points $(-1, 4)$ and $(-2, 6)$
Answer: $(-1, 4)$ and $(-2, 6)$

10 Simplify using double angle formulae (All boards)

Simplify $\frac{\sin 2x}{\sin x}$,
giving your answer as a single trigonometric term.

$\sin 2x = 2 \sin x \cos x$

so the expression is $\frac{2 \sin x \cos x}{\sin x} = 2 \cos x$

Answer: $2 \cos x$