Day 59 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = 3t^2 - 4t + 3$ m s^{-1}.
Find the distance moved between t = 0 and t = 4. (The particle does not change direction.)

$s = \int_0^4 (3t^2 - 4t + 3)\,dt$

$= [\,t^3 - 2t^2 + 3t\,]$ from t = 0 to t = 4

$= 44 - 0$

Answer: 44 m

2 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 4x - 2y - 27 = 0$.
Find the coordinates of its centre and its radius.

Complete the square in x and y:

$(x - 2)^2 - 4 + (y - 1)^2 - 1 - 27 = 0$

$(x - 2)^2 + (y - 1)^2 = 32$

Centre (2, 1), radius $4\sqrt{2}$

Answer: centre (2, 1), radius $4\sqrt{2}$

3 Quotient rule (All boards)

Given that $y = \frac{x^2 + 3}{x + 2}$,

find $\frac{dy}{dx}$, and its value when x = 1.

Quotient rule with $u = x^2 + 3$, $v = x + 2$

$u' = 2x$, $v' = 1$

$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{2x(x + 2) - x^2 - 3}{(x + 2)^2}$

When x = 1: $\frac{dy}{dx} = \frac{2}{9}$

Answer: $\frac{dy}{dx} = \frac{2x(x + 2) - x^2 - 3}{(x + 2)^2}$; value $\frac{2}{9}$

4 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{-3}{x - 2} + 3$.

(a) Write down the asymptotes.
(b) Find where the curve crosses the axes.

(a) The denominator is zero when x = 2, and $y \to 3$ as $x \to \pm\infty$
Asymptotes: x = 2 and y = 3

(b) x = 0: $y = \frac{3}{2} + 3 = \frac{9}{2}$

y = 0: $\frac{-3}{x - 2} = -3$, so $x - 2 = 1$, $x = 3$

Answer: (a) x = 2, y = 3 (b) $(0, \frac{9}{2})$ and (3, 0)

5 Two particles over a pulley (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 1 kg and 4 kg hang on a light string over a smooth fixed pulley, as shown, and are released from rest.
Take g = 9.8 m s^{-2}.
Find the acceleration and the tension in the string.

Heavier: $4g - T = 4a$

Lighter: $T - 1g = 1a$

Add: $3g = 5a$, so $a = 5.88$

$T = 1(9.8 + 5.88)$

Answer: a = 5.88 m s^{-2}; T = 15.68 N

6 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $2\sin^2 x - \cos x - 2 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use $\sin^2 x = 1 - \cos^2 x$: $-2\cos^2 x - \cos x = 0$, i.e. $2\cos^2 x + \cos x = 0$

$(\cos x)(2\cos x + 1) = 0$, so $\cos x = 0$ or $-\frac{1}{2}$

$\cos x = 0$: $x = 90.0°, 270.0°$

$\cos x = -\frac{1}{2}$: $x = 120.0°, 240.0°$

Answer: x = 90.0°, 120.0°, 240.0°, 270.0°

7 Poisson distribution (CIE)

X ~ Po(4.5), modelling the number of flaws per metre of cloth.
Find, to 3 significant figures,
(a) P(X = 6)
(b) $P(X \leq 6)$

(a) $P(X = 6) = e^{-4.5} \times \frac{4.5^6}{6!} = 0.128$

(b) $P(X \leq 6) = P(X = 0) + ... + P(X = 6) = 0.831$

Answer: (a) 0.128 (b) 0.831

8 Double angle equations (All boards)

Solve $\sin 2x = 5\cos x$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $2\sin x \cos x - 5\cos x = 0$
$\cos x(2\sin x - 5) = 0$
$\cos x = 0$: $x = 90.0°, 270.0°$

$\sin x = \frac{5}{2}$ has no solutions

Answer: x = 90.0°, 270.0°

9 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-4, 8)$ and $(-5, 1)$.
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= (\frac{-4 + (-5)}{2}, \frac{8 + 1}{2}) = (-\frac{9}{2}, \frac{9}{2})$

(b) Gradient $= \frac{-7}{-1} = 7$

$y - 8 = 7(x + 4)$
Multiply out and collect terms: $7x - y + 36 = 0$

Answer: (a) $(-\frac{9}{2}, \frac{9}{2})$ (b) $7x - y + 36 = 0$

10 Poisson: changing the interval (CIE)

Customers arrive at a shop at random at an average rate of 3 per hour.
Find the probability that in 30 minutes there are exactly 2 (3 s.f.).

Rate in 30 minutes: $\lambda = 3 \times \frac{30}{60} = 1.5$

X ~ Po(1.5)

$P(X = 2) = e^{-1.5} \times \frac{1.5^2}{2!} = 0.251$

Answer: 0.251