Day 58 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Quotient rule (All boards)

Find the x-coordinate(s) of the stationary point(s) of the curve $y = \frac{x}{x^2 + 5}$.

$\frac{dy}{dx} = \frac{5 - x^2}{(x^2 + 5)^2}$

Stationary points where $\frac{dy}{dx} = 0$, i.e. the numerator is zero

Numerator: $5 - x^2 = 0$

$x = -\sqrt{5}$ or $x = \sqrt{5}$

Answer: $x = -\sqrt{5}$, $x = \sqrt{5}$

2 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $-2x^2 + 24x - 69$ in the form $a(x + p)^2 + q$.

(b) Write down the coordinates of the maximum point of $y = -2x^2 + 24x - 69$.

Take out the factor -2: $-2x^2 + 24x - 69 = -2(x^2 - 12x) - 69$

$= -2[(x - 6)^2 - 36] - 69$

$= -2(x - 6)^2 + 3$

$(x - 6)^2 \geq 0$ with equality at $x = 6$, so the maximum is 3 at $x = 6$.

Answer: (a) $-2(x - 6)^2 + 3$ (b) (6, 3)

3 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 4 m and mass 4 kg hangs horizontally from two vertical strings attached 0.5 m from A and 0.5 m from B.

A particle of mass 5 kg is attached 2 m from A. Take $g = 9.8$ m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(3) = 4\,g(1.5) + 5\,g(1.5)$

$T_2 = 44.1$

$T_1 + T_2 = 9\,g$ so $T_1 = 88.2 - 44.1$

Answer: Near A: 44.1 N; near B: 44.1 N

4 Quartiles and outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Ordered data, waiting times (min) of 7 patients:
16, 23, 27, 31, 35, 37, 40.

Find Q_1, Q_3, the IQR and any outliers (more than 1.5 × IQR beyond a quartile).

$Q_1 = 23$, $Q_3 = 37$, IQR = 14

Lower limit = $23 - 1.5 \times 14 = 2$

Upper limit = $37 + 1.5 \times 14 = 58$

No value lies outside these limits.

Answer: $Q_1 = 23$, $Q_3 = 37$, IQR = 14; no outliers

5 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The height of the tide in a harbour is modelled by $H = 7 + 6\sin(\frac{\pi t}{6})$, where t is in hours after midnight.

Find (a) the greatest and least values, (b) the values of t with $0 \leq t \leq 12$ when it equals 10 m.

(a) sin ranges from -1 to 1: greatest $7 + 6 = 13$, least $7 - 6 = 1$

(b) $7 + 6\sin(\frac{\pi t}{6}) = 10$, so $\sin(\frac{\pi t}{6}) = \frac{1}{2}$

$\frac{\pi t}{6} = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, so $t = 1$ or 5

Answer: (a) 13 m, 1 m (b) t = 1 and 5

6 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x + 4)x(x + 3) \leq 0$.

Critical values: $x = -4$, $x = -3$, $x = 0$

The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.

Reading from the sketch: $x \leq -4$ or $-3 \leq x \leq 0$

Answer: $x \leq -4$ or $-3 \leq x \leq 0$

7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 6 ml. Set mean 200 ml; 40 bottles have mean 197.7 ml. Test at the 5% level whether the true mean differs from 200.

$H_0: \mu = 200$, $H_1: \mu \neq 200$. Under H_0, $\overline{X} \sim N(200, \frac{6^2}{40})$

$z = \frac{197.7 - 200}{6/\sqrt{40}} = -2.424$

$P(Z < -2.424) = 0.0077 < 0.025$

Reject H_0: there is sufficient evidence that the mean has changed

Answer: reject H_0: sufficient evidence of a change in the mean

8 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{7x + 10}{(x + 1)(x + 2)}$ in partial fractions.

(b) Hence evaluate $\int_0^1 \frac{7x + 10}{(x + 1)(x + 2)}\,dx$, giving the answer as a single logarithm.

(a) $\frac{7x + 10}{(x + 1)(x + 2)} \equiv \frac{A}{x + 1} + \frac{B}{x + 2}$, so $7x + 10 \equiv A(x + 2) + B(x + 1)$

$x = -1$: $A = 3$; $x = -2$: $B = 4$

(b) $\int_0^1 \left(\frac{3}{x + 1} + \frac{4}{x + 2}\right) dx = [3\ln(x + 1) + 4\ln(x + 2)]_0^1$

$= 3\ln\frac{2}{1} + 4\ln\frac{3}{2} = \ln\left(\frac{81}{2}\right)$

Answer: (a) $A = 3$, $B = 4$ (b) $\ln\left(\frac{81}{2}\right)$

9 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{8 - 4\sqrt{2}}{2 - 2\sqrt{2}}$ in the form $p + q\sqrt{2}$, where p and q are integers.

Multiply top and bottom by the conjugate $(2 + 2\sqrt{2})$:

Denominator: $(2 - 2\sqrt{2})(2 + 2\sqrt{2}) = 2^2 - 2^2 \times 2 = -4$

Numerator: $(8 - 4\sqrt{2})(2 + 2\sqrt{2}) = 8\sqrt{2}$

So the fraction $= \frac{8\sqrt{2}}{-4} = -2\sqrt{2}$

Answer: $-2\sqrt{2}$

10 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 4 kg is pulled along rough horizontal ground by a horizontal force of 21.6 N. It accelerates at 0.5 m s^{-2}. Take $g = 9.8$ m s^{-2}.

Find the coefficient of friction between the sledge and the ground.

$R = mg = 4 \times 9.8 = 39.2$

Resultant: $21.6 - F = 4 \times 0.5$, so $F = 19.6$

$F = \mu R$: $\mu = 19.6 \div 39.2$

Answer: 0.5