Day 57 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplify using double angle formulae (All boards)

Simplify $\frac{1 - \cos 2x}{1 + \cos 2x}$,

giving your answer as a single trigonometric term.

$1 - \cos 2x = 2\sin^2 x$ and $1 + \cos 2x = 2\cos^2 x$

so the expression is $\frac{2\sin^2 x}{2\cos^2 x} = \tan^2 x$

Answer: $\tan^2 x$

2 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 4 s, (b) the distance between the stations.

(a) a = gradient = 8 ÷ 4

(b) Distance = area of trapezium = ½ × (28 + 20) × 8

Answer: (a) 2 m s^{-2} (b) 192 m

3 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $32^{\frac{3}{5}}$.

$32 = 2^5$, so $32^{\frac{3}{5}} = (2^5)^{\frac{3}{5}} = 2^3$

$= 8$

Answer: 8

4 Binomial mean and variance (All boards)

X has a binomial distribution with mean 9 and variance 6.3.
(a) Find n and p.
(b) Find P(X = 9) to 3 significant figures.

(a) $np = 9$ and $np(1 - p) = 6.3$

$1 - p = 6.3 \div 9 = 0.7$, so $p = 0.3$ and $n = 9 \div 0.3 = 30$

(b) $P(X = 9) = {}^{30}C_9 \times 0.3^9 \times 0.7^{21} = 0.157$

Answer: (a) n = 30, p = 0.3 (b) 0.157

5 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(−6, 8) and B(2, −4), and its centre lies on the x-axis. Find the centre and the equation of the circle.

Let the centre be (h, 0). Then $CA^2 = CB^2$:

$(h + 6)^2 + 64 = (h - 2)^2 + 16$

Expanding and cancelling h^2: $12h + 100 = -4h + 20$

$h = -5$

$r^2 = CA^2 = 65$

Answer: centre (−5, 0), $(x + 5)^2 + y^2 = 65$

6 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), find an approximation of the
form $1 + p\theta + q\theta^2$ for
$\sin 3\theta + \cos 4\theta + \tan \theta$.

$\sin 3\theta \approx 3\theta$, $\cos 4\theta \approx 1 - \frac{16\theta^2}{2}$, $\tan \theta \approx \theta$

Sum $\approx 1 + 3\theta - \frac{16\theta^2}{2} + \theta$

$\approx 1 + 4\theta - 8\theta^2$

Answer: p = 4, q = −8

7 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf $f(x) = kx^2$ for $0 \le x \le 3$, and 0 otherwise.
(a) Find the value of k.
(b) Find P(X > 2). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^3/3]$ from 0 to 3 $= k \times \frac{3^3}{3} = 1$, so $k = \frac{1}{9}$

(b) $P(X > 2) = \int_2^3 \frac{1}{9} x^2\,dx = 1 - \frac{2^3}{3^3} = \frac{19}{27}$

(c) $E(X) = \int_0^3 x\,f(x)\,dx = \frac{1}{9} \times \frac{3^4}{4} = \frac{9}{4}$

Answer: (a) $k = \frac{1}{9}$ (b) $\frac{19}{27}$ (c) $\frac{9}{4}$

8 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 15°C: $\frac{dT}{dt} = -k(T - 15)$. Initially T = 95 and after 6 minutes T = 55.
Find k to 3 s.f. and the time to cool to 25°C, to 3 s.f.

$\frac{1}{T - 15}\,dT = -k\,dt$, so $\ln(T - 15) = -kt + c$, i.e. $T = 15 + 80e^{-kt}$

$t = 6$: $55 = 15 + 80e^{-6k}$, so $e^{-6k} = \frac{1}{2}$

$k = \frac{1}{6} \ln \frac{80}{40} = 0.1155$

$25 = 15 + 80e^{-kt}$: $t = \frac{1}{k} \ln \frac{80}{10} = 18.00$

Answer: k = 0.116, t = 18.0 minutes

9 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $2^{2x} + 2^x - 6 = 0$.

Let $y = 2^x$. Then $2^{2x} = y^2$, so $y^2 + y - 6 = 0$
$(y - 2)(y + 3) = 0$
$y = 2$ or $y = -3$
$2^x > 0$ for all x, so $y = -3$ is rejected. $2^x = 2$ gives $x = 1$.
Answer: x = 1

10 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cricket ball is projected from ground level with initial velocity components 7 m s^{-1} horizontally and 29.4 m s^{-1} vertically upwards.
Take g = 9.8 m s^{-2}.
Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 29.4^2 - 2 \times 9.8 \times H$, $H = 864.36 \div 19.6$
(b) At the top the vertical velocity is zero, so the speed is the horizontal component.
Answer: (a) 44.1 m (b) 7 m s^{-1}