Day 56 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Quartiles and outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Ordered data, texts sent in a day by 11 students:
12, 12, 13, 19, 25, 27, 30, 31, 37, 38, 39.
Find Q_1, Q_3, the IQR and any outliers (more than 1.5 × IQR beyond a quartile).

$Q_1 = 13$, $Q_3 = 37$, IQR = 24
Lower limit = $13 - 1.5 \times 24 = -23$
Upper limit = $37 + 1.5 \times 24 = 73$
No value lies outside these limits.
Answer: $Q_1 = 13$, $Q_3 = 37$, IQR = 24; no outliers

2 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 1 : 2, and N lies on OB with ON : NB = 1 : 3.
Find OM and NM in terms of a and b.

$\text{AB} = \text{b} - \text{a}$, so $\text{AM} = \frac{1}{3}(\text{b} - \text{a})$

$\text{OM} = \text{a} + \text{AM} = \frac{2\text{a}}{3} + \frac{\text{b}}{3}$

$\text{ON} = \frac{\text{b}}{4}$, so $\text{NM} = \text{OM} - \text{ON} = \frac{2\text{a}}{3} + \frac{\text{b}}{12}$

Answer: $\text{OM} = \frac{2\text{a}}{3} + \frac{\text{b}}{3}$, $\text{NM} = \frac{2\text{a}}{3} + \frac{\text{b}}{12}$

3 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = t^2 + 2t + 6$ m s^{-1}.
Find the distance moved between t = 2 and t = 5. (The particle does not change direction.)

$s = \int_2^5 (t^2 + 2t + 6)\,dt$

$= \left[\frac{1}{3}t^3 + t^2 + 6t\right]$ from t = 2 to t = 5

$= 290/3 - 56/3$

Answer: 78 m

4 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + 1}\,dx$,

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{\cos x}{\sin x + 1}\,dx = \ln(\sin x + 1)$

$= [\ln(\sin x + 1)]_0^{\frac{\pi}{2}}$

$= \ln 2$

Answer: ln 2

5 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 - 2x + 3$ for $x \geq 1$.
Find $f^{-1}(x)$ and state its domain.

Complete the square: $f(x) = (x - 1)^2 + 2$
$y = (x - 1)^2 + 2$, so $(x - 1)^2 = y - 2$
As $x - 1 \geq 0$ take the positive root: $x = 1 + \sqrt{y - 2}$
As $x - 1 \geq 0$, take the positive root: $x = 1 + \sqrt{y - 2}$
Answer: $f^{-1}(x) = 1 + \sqrt{x - 2}$, domain $x \geq 2$

6 Total probability and Bayes (All boards)

R: it rains; L: Ali is late.
P(R) = 0.6, P(L | R) = 0.5, P(L | R') = 0.1.
(a) Find P(L).
(b) Find P(R | L).

(a) P(L) = 0.6 × 0.5 + 0.4 × 0.1 = 0.3 + 0.04 = 0.34

(b) P(R | L) = $\frac{0.6 \times 0.5}{0.34} = \frac{0.3}{0.34} = 0.882$

Answer: (a) 0.34 (b) $\frac{15}{17}$

7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that cos θ + 3 sin θ ≡ $\sqrt{10}$ cos(θ − 71.6°),
solve cos θ + 3 sin θ = 0.5 for 0 ≤ θ ≤ 360°. (1 d.p.)

$\sqrt{10}$ cos(θ − 71.6°) = 0.5, so cos(θ − 71.6°) = $\frac{0.5}{\sqrt{10}}$

θ − 71.6° = ±80.9°
θ = 71.6° + 80.9° = 152.5°
θ = 71.6° − 80.9° = −9.3°, so θ = 350.7°
Answer: θ = 152.5°, 350.7°

8 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is kicked from horizontal ground with speed 49 m s^{-1} at an angle θ above the horizontal, where tan θ = $\frac{4}{3}$.

Take g = 9.8 m s^{-2}.
Find (a) the time of flight, (b) the horizontal range.

cos θ = 3/5, sin θ = 4/5: horizontal 29.4, vertical 39.2
(a) 0 = 39.2 t − 4.9t^2, t = 39.2 ÷ 4.9
(b) Range = 29.4 × 8
Answer: (a) 8 s (b) 235.2 m

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers
m and n such that 18m + 24n = 64.

Assume there are integers m and n with 18m + 24n = 64.
18m + 24n = 6(3m + 4n).
So the left-hand side is a multiple of 6.
But 64 = 6 × 10 + 4 is not a multiple of 6.
This is a contradiction, so no such integers exist.
Answer: no integer solutions (LHS is a multiple of 6, 64 is not)

10 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 2x^3 − 2x^2 + kx + 18.
Given that (x + 3) is a factor of f(x),
(a) find the value of k,
(b) factorise f(x) completely.

f(−3) = 0: −54 − 18 − 3k + 18 = 0
k = −18
f(x) = (x + 3)(2x^2 − 8x + 6)
= 2(x − 3)(x − 1)(x + 3)
Answer: (a) k = −18 (b) f(x) = 2(x − 3)(x − 1)(x + 3)