Day 55 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-5x + 7 < 3x - 7$.

$-8x < -14$

Divide by -8 (reversing the inequality): $x > \frac{7}{4}$

Answer: $x > \frac{7}{4}$

2 Resultant of perpendicular forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two forces of magnitude 7 N and 24 N act at right angles on a particle.
Find the magnitude of the resultant, and the angle it makes with the 7 N force, to 1 d.p.

Magnitude = $\sqrt{(7^2 + 24^2)} = \sqrt{625}$
$\tan \alpha = 24 \div 7$
Answer: 25 N at 73.7°

3 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 6 cm and angle 240°.
Find, in terms of π, the arc length and the area of the sector.

$\theta = 240° = \frac{4\pi}{3}$ radians

Arc length = $r\theta = 8\pi$ cm

Area = $\frac{1}{2}r^2\theta = 24\pi$ cm^2

Answer: arc 8π cm, area 24π cm^2

4 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 4x + 8y + 10 = 0$.
Find the coordinates of its centre and its radius.

Complete the square in x and y:
$(x - 2)^2 - 4 + (y + 4)^2 - 16 + 10 = 0$
$(x - 2)^2 + (y + 4)^2 = 10$
Centre $(2, -4)$, radius $\sqrt{10}$
Answer: centre $(2, -4)$, radius $\sqrt{10}$

5 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix} -2 \\ -1 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -10 \\ -9 \\ 9 \end{pmatrix}$.

Find AB and the distance AB.

$\mathrm{AB} = \mathrm{OB} - \mathrm{OA} = \begin{pmatrix} -8 \\ -8 \\ 4 \end{pmatrix}$

$|\mathrm{AB}| = \sqrt{8^2 + 8^2 + 4^2} = \sqrt{144} = 12$

Answer: AB = $\begin{pmatrix} \mathbf{-8} \\ \mathbf{-8} \\ \mathbf{4} \end{pmatrix}$, distance 12

6 Total probability and Bayes (All boards)

R: has the condition; L: test is positive.
P(R) = 0.2, P(L | R) = 0.5, P(L | R') = 0.2.
(a) Find P(L).
(b) Find P(R | L).

(a) $P(L) = 0.2 \times 0.5 + 0.8 \times 0.2 = 0.1 + 0.16 = 0.26$

(b) $P(R \mid L) = \frac{0.2 \times 0.5}{0.26} = \frac{0.1}{0.26} = 0.385$

Answer: (a) 0.26 (b) $\frac{5}{13}$

7 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $-2x^2 + 16x - 29$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the maximum point of $y = -2x^2 + 16x - 29$.

Take out the factor -2: $-2x^2 + 16x - 29 = -2(x^2 - 8x) - 29$
$= -2[(x - 4)^2 - 16] - 29$
$= -2(x - 4)^2 + 3$
$(x - 4)^2 \geq 0$ with equality at $x = 4$, so the maximum is 3 at $x = 4$.

Answer: (a) $-2(x - 4)^2 + 3$ (b) (4, 3)

8 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{3x + 12}{(x + 2)(x + 5)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{3x + 12}{(x + 2)(x + 5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{3x + 12}{(x + 2)(x + 5)} \equiv \frac{A}{x + 2} + \frac{B}{x + 5}$, so $3x + 12 \equiv A(x + 5) + B(x + 2)$
$x = -2$: $A = 2$; $x = -5$: $B = 1$

(b) $\int_0^4 \left(\frac{2}{x + 2} + \frac{1}{x + 5}\right) dx = [2\ln(x + 2) + \ln(x + 5)]_0^4$
$= 2\ln\frac{6}{2} + \ln\frac{9}{5} = \ln\left(\frac{81}{5}\right)$

Answer: (a) A = 2, B = 1 (b) $\ln\left(\frac{81}{5}\right)$

9 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (6 kg) and B (9 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 0.5 m above the ground, then released.
Take $g = 9.8$ m s^{-2}.
Find the speed with which B hits the ground.

$a = (9 - 6)g \div (9 + 6) = 1.96$
$v^2 = 0 + 2 \times 1.96 \times 0.5 = 1.96$
$v = 1.4$

Answer: 1.4 m s^{-1}

10 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.
P(X < 55) = 0.1587 and P(X > 65) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$
$55 = \mu + (-0.9998)\sigma$ and $65 = \mu + 0.9998\sigma$
Subtract: $10 = 2.000\,\sigma$, so $\sigma = 5.00$
$\mu = 55 - (-0.9998)(5.001) = 60.0$

Answer: $\mu = 60.0$, $\sigma = 5.00$