Day 54 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x - 3 \leq 3x - 9$.

$-4x \leq -6$

Divide by -4 (reversing the inequality): $x \geq \frac{3}{2}$

Answer: $x \geq \frac{3}{2}$

2 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate

$\int_1^3 x \ln(x + 1)\, dx$,

giving your answer to 3 significant figures.

$h = \frac{3 - 1}{4} = 0.5$

x: 1, 1.5, 2, 2.5, 3
y: 0.6931, 1.374, 2.197, 3.132, 4.159

Area $\approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)]$

$= \frac{0.5}{2} \times [0.6931 + 4.159 + 2 \times 6.704] = 4.565$

Answer: 4.56

3 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "If $x^2 > 4$ then $x > 2$."
Give a counter-example to show the statement is false.

$x^2 > 4$ is also true for negative x with $x < -2$.
Take $x = -3$: $x^2 = 9 > 4$, but -3 is not greater than 2.
Answer: $x = -3$

4 Area under a curve (All boards)

The region R is bounded by the axes, the line $x = \ln 3$
and the curve $y = 2e^x + 3$. Find the exact area of R.

Area $= \int_0^{\ln 3} (2e^x + 3)\, dx = [3x + 2e^x]_0^{\ln 3}$
$= \ln 27 + 4$
Answer: $\ln 27 + 4$

5 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = 3t^3 + 3t^2 - t$ metres.
Find its velocity and acceleration when $t = 2$.

$v = ds/dt = 9t^2 + 6t - 1$
$a = dv/dt = 18t + 6$
$t = 2$: $v = 47$, $a = 42$
Answer: $v = 47$ m s^{-1}, $a = 42$ m s^{-2}

6 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = 3$ and θ is acute,
find the exact value of $\operatorname{cosec}\theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
$\tan\theta = 3$, so $\sec^2\theta = 1 + 9 = 10$

$\cot\theta = \frac{1}{3}$, so $\operatorname{cosec}^2\theta = \frac{10}{9}$ and $\operatorname{cosec}\theta = \frac{\sqrt{10}}{3}$

Answer: $\frac{\sqrt{10}}{3}$

7 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $12x - 5y = k$ is a tangent to the circle
$(x - 1)^2 + (y + 1)^2 = 169$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to $(12, -5)$.
A unit step in this direction is $(12, -5)/13$, so the points of contact are
$(1, -1) \pm 13 \times (12, -5)/13 = (1, -1) \pm (12, -5)$
T = $(13, -6)$ or $(-11, 4)$, and $k = 12x - 5y$ gives:
k = 186 or k = −152
Answer: k = 186 or k = −152

8 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.2. It is tossed 40 times and lands heads 14 times.
Test, at the 5% significance level, whether the probability of heads is different from 0.2.

H_0: p = 0.2, H_1: $p \neq 0.2$. Under H_0, $X \sim B(40, 0.2)$
$P(X \geq 14) = 1 - P(X \leq 13) = 0.0194$
Two-tailed: compare with 0.025. 0.0194 < 0.025, so reject H_0
There is sufficient evidence that the probability of heads is different from 0.2
Answer: reject H_0: sufficient evidence that the probability of heads is different from 0.2

9 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 3 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$.

The coefficient of friction is 0.125. Take g = 9.8 m s^{-2}. Find the acceleration down the plane, and the distance moved in 2 s.

$\cos \alpha = 4/5$, $\sin \alpha = 3/5$
$R = mg \cos \alpha$, $F = \mu R$
$ma = mg \sin \alpha - \mu mg \cos \alpha$, so $a = 9.8(3/5 - 0.125 \times 4/5)$
$s = \frac{1}{2}at^2 = \frac{1}{2} \times 4.9 \times 2^2$
Answer: 4.9 m s^{-2}; 9.8 m

10 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.
P(X < 88) = 0.1587 and P(X > 112) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$
$88 = \mu + (-0.9998)\sigma$ and $112 = \mu + 0.9998\sigma$
Subtract: $24 = 2.000\ \sigma$, so $\sigma = 12.0$
$\mu = 88 - (-0.9998)(12.00) = 100$
Answer: μ = 100, σ = 12.0