Day 53 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A remote-control car moves with constant velocity $(-30\mathbf{i} + 40\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north. Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed $= \sqrt{(-30^2 + 40^2)} = 50$

$\tan\theta = 40 \div 30$

Answer: 50 m s^{-1}; 53.1°

2 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 14 pairs of values, $\bar{x} = 8$, $\bar{y} = 10$, $S_{xx} = 100$ and $S_{xy} = 100$.

(a) Find the regression line of y on x in the form $y = a + bx$.

(b) Estimate y when $x = 12$.

(a) $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{100}{100} = 1$

$a = \bar{y} - b\bar{x} = 10 - 1 \times 8 = 2$

$y = 2 + x$

(b) $y = 2 + 1 \times 12 = 14$

Answer: y = 2 + x; y = 14

3 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $8^{x-3} = 2^{4x-5}$.

Write both sides as powers of 2: $2^{3(x-3)} = 2^{4x-5}$

Equate the indices: $3x - 9 = 4x - 5$

$-x = 4$

$x = -4$

Answer: x = −4

4 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the trapezium rule with 5 strips.

Use it to estimate $\displaystyle\int_0^1 \frac{1}{\sqrt{1+x^3}}\,dx$, to 3 s.f.

$h = \dfrac{1-0}{5} = 0.2$

x: 0, 0.2, 0.4, 0.6, 0.8, 1

y: 1.000, 0.9960, 0.9695, 0.9068, 0.8133, 0.7071

Area $\approx \dfrac{h}{2}[y_0 + y_5 + 2(y_1 + \ldots + y_4)]$

$= \dfrac{0.2}{2} \times [1.000 + 0.7071 + 2 \times 3.686] = 0.9078$

Answer: 0.908

5 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \dfrac{-2}{x-4} + 2$.

(a) Write down the asymptotes.

(b) Find where the curve crosses the axes.

(a) The denominator is zero when $x = 4$, and $y \to 2$ as $x \to \pm\infty$

Asymptotes: $x = 4$ and $y = 2$

(b) $x = 0$: $y = \dfrac{1}{2} + 2 = \dfrac{5}{2}$

$y = 0$: $\dfrac{-2}{x-4} = -2$, so $x - 4 = 1$, $x = 5$

Answer: (a) x = 4, y = 2 (b) $(0, \frac{5}{2})$ and (5, 0)

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6 Grouped data: mean and median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises 44 measurements of m.
(a) Estimate the mean.
(b) Estimate the median using linear interpolation.

(a) Using mid-points: $\Sigma fx = 1470$, so mean = $1470 \div 44 = 33.4$

(b) $\frac{n}{2} = 22$. Cumulative frequency before the median class = 19

Median class 30 to 35. Median = $30 + \frac{3}{3} \times 5 = 35.0$

Answer: mean ≈ 33.4, median ≈ 35.0

7 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = x(4 - x)$, the x-axis
and the lines $x = 0$ and $x = 4$.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_0^4 y^2 \, dx = \pi \int_0^4 (x^4 + 16x^2 - 8x^3) \, dx$

$= \pi[\frac{x^5}{5} + \frac{16x^3}{3} - 2x^4]_0^4$

$= \frac{512\pi}{15}$

Answer: $\frac{512\pi}{15}$

8 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = t^3 + 2t^2$ metres.
Find (a) the average velocity over the first 4 seconds, (b) the velocity at t = 4.

(a) $s(4) = 96$, so average velocity = $96 \div 4$

(b) $v = ds/dt = 3t^2 + 4t$, $v(4) = 64$

Answer: (a) 24 m s^{-1} (b) 64 m s^{-1}

9 Geometric sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 9 and sum to infinity 6.
Find the common ratio and the value of the 4th term.

$S_\infty = \frac{a}{1 - r}$: $6 = \frac{9}{1 - r}$

$1 - r = \frac{9}{6}$, so $r = -\frac{1}{2}$

$u_4 = ar^3 = 9 \times (-\frac{1}{2})^3 = -\frac{9}{8}$

Answer: $r = -\frac{1}{2}$, $u_4 = -\frac{9}{8}$

10 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a} \cdot \mathbf{b} = (-3)(-1) + (1)(0) + (2)(5) = 13$

$|\mathbf{a}| = \sqrt{14}$, $|\mathbf{b}| = \sqrt{26}$

$\cos\theta = \frac{13}{\sqrt{14}\sqrt{26}} = 0.6814$

$\theta = 47.0°$

Answer: 47.0°