Day 52 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = t^3 + 3t^2 + 3t$ metres.
Find its velocity and acceleration when t = 4.

$v = ds/dt = 3t^2 + 6t + 3$
$a = dv/dt = 6t + 6$
$t = 4$: $v = 75$, $a = 30$
Answer: $v = 75$ m s^{-1}, $a = 30$ m s^{-2}

2 Binomial probabilities (All boards)

$X \sim B(28, 0.1)$. Find, to 3 significant figures,
(a) $P(X > 2)$
(b) $P(X \leq 2)$

(a) $P(X > 2) = 1 - P(X \leq 2)$
(a) $= 0.541$
(b) $P(X \leq 2)$ from the cumulative binomial function
(b) $= 0.459$
Answer: (a) 0.541 (b) 0.459

3 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-5x - 13}{(x + 2)(x + 3)}$ in partial fractions.

(b) Hence evaluate $\int_0^1 \frac{-5x - 13}{(x + 2)(x + 3)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-5x - 13}{(x + 2)(x + 3)} \equiv \frac{A}{x + 2} + \frac{B}{x + 3}$, so $-5x - 13 \equiv A(x + 3) + B(x + 2)$
$x = -2$: $A = -3$; $x = -3$: $B = -2$
(b) $\int_0^1 \left(\frac{-3}{x + 2} + \frac{-2}{x + 3}\right) dx = [-3\ln(x + 2) - 2\ln(x + 3)]_0^1$
$= -3\ln\frac{3}{2} - 2\ln\frac{4}{3} = \ln\left(\frac{1}{6}\right)$

Answer: (a) $A = -3$, $B = -2$ (b) $\ln\left(\frac{1}{6}\right)$

4 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 23 students.
Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 23$, $\Sigma fx = 67$, $\Sigma fx^2 = 219$
Mean $= \frac{67}{23} = 2.91$
Variance $= \frac{219}{23} - (2.9130)^2 = 1.036$
Standard deviation $= 1.02$
Answer: mean 2.91, standard deviation 1.02

5 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 + 2x - 5$. Use the Newton-Raphson method with
$x_0 = 1$ to find x_1 and x_2, to 4 d.p.

$f'(x) = 3x^2 + 2$
$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
$x_1 = 1 - \frac{-2.000}{5.000} = 1.4000$
$x_2 = 1.4000 - \frac{0.5440}{7.880} = 1.3310$
Answer: $x_1 = 1.4000$, $x_2 = 1.3310$

6 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x + 4)^2 + (y + 3)^2 = 25$.
The point P(−1, 1) lies on C.
Find an equation of the tangent to C at P, in the form $ax + by + c = 0$ with integer a, b, c.

Centre of C = (−4, −3). Gradient of radius CP = $\frac{4}{3}$

The tangent is perpendicular to the radius: gradient = $-\frac{3}{4}$

$y - 1 = -\frac{3}{4}(x + 1)$

Multiply out and collect terms: $3x + 4y - 1 = 0$

Answer: $3x + 4y - 1 = 0$

7 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 4 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{4}{3}$.

Take g = 9.8 m s^{-2}.
Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{4}{5} = T_2 \times \frac{3}{5}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 39.2$

From the first: $T_1 = \frac{3}{4} T_2$; into the second: $T_2 \times \frac{5}{4} = 39.2$

Answer: T_1 = 23.52 N (at α); T_2 = 31.36 N (at β)

8 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $6\cos\theta + 8\sin\theta \equiv 10\cos(\theta - 53.1°)$,
solve $6\cos\theta + 8\sin\theta = -5$ for $0 \le \theta \le 360°$. (1 d.p.)

$10\cos(\theta - 53.1°) = -5$, so $\cos(\theta - 53.1°) = \frac{-5}{10}$

$\theta - 53.1° = \pm 120.0°$
$\theta = 53.1° + 120.0° = 173.1°$
$\theta = 53.1° - 120.0° = -66.9°$, so $\theta = 293.1°$
Answer: θ = 173.1°, 293.1°

9 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|3x + 7| < 5$.

$|3x + 7| < 5$ means $-5 < 3x + 7 < 5$

Subtract 7 and divide by 3: $-4 < x < -\frac{2}{3}$

Answer: $-4 < x < -\frac{2}{3}$

10 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $5x + 12y = k$ is a tangent to the circle
$(x + 1)^2 + (y - 3)^2 = 169$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (5, 12).
A unit step in this direction is (5, 12)/13, so the points of contact are
(−1, 3) ± 13 × (5, 12)/13 = (−1, 3) ± (5, 12)
T = (4, 15) or (−6, −9), and $k = 5x + 12y$ gives:
k = 200 or k = −138
Answer: k = 200 or k = −138