Day 51 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of sec 60°,
giving your answer with a rational denominator.

$\sec 60° = \frac{1}{\cos 60°}$

$\cos 60° = \frac{1}{2}$, so $\sec 60° = \frac{1}{\frac{1}{2}}$

Answer: 2

2 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.
(a) Describe the correlation.
(b) Why should $y = 12 - x$ not be used for $x = 20$?

(a) The points lie loosely around a straight line falling: weak negative correlation
(b) The data cover $x = 1$ to 10; $x = 20$ is outside this range
so using the line would be extrapolation and may be unreliable
Answer: (a) weak negative correlation (b) extrapolation outside the range of the data

3 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(4x^3)^2 \times 3x^2}{3x^4}$.

$(4x^3)^2 = 16x^6$

Numerator $= 16 \times 3x^8 = 48x^8$

Divide: $\frac{48}{3} = 16$ and $x^8 \div x^4 = x^4$

Answer: $16x^4$

4 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 2e^x - 8 = 0$,
giving any answers as exact values.

Let $y = e^x$: $y^2 - 2y - 8 = 0$
$(y + 2)(y - 4) = 0$
$e^x > 0$, so $y = -2$ is rejected; $e^x = 4$
$x = \ln 4$
Answer: $x = \ln 4$

5 Particle on a table joined to a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 2 kg lies on a smooth horizontal table. It is attached by a light string over a smooth pulley at the edge of the table to a particle of mass 6 kg hanging freely.
Take $g = 9.8$ m s^{-2}.
The system is released from rest. Find the acceleration and the tension in the string.

Hanging: $6g - T = 6a$
Table: $T = 2a$
$6g = 8a$, so $a = 7.35$
$T = 2 \times 7.35$
Answer: $a = 7.35$ m s^{-2}; $T = 14.7$ N

6 Tree diagram (All boards)

The tree diagram shows probabilities for event E, followed by event F.
Find the probability that exactly one of E and F occurs.

P(E and not F) $= 0.4 \times 0.8 = 0.32$
P(not E and F) $= 0.6 \times 0.6 = 0.36$
Exactly one: $0.32 + 0.36 = 0.68$
Answer: 0.68

7 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $x + 2y = -5$ and line l_2 has equation $4x + y = 8$.

The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (3, −4)

l_1 meets y = 0 at x = −5, and l_2 meets y = 0 at x = 2

Base = 7, height = 4

Area = $\frac{1}{2}$ × base × height = 14

Answer: 14

8 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 4 m and mass 20 kg rests horizontally on supports at C and D, where AC = 1 m and DB = 1.5 m.

A girl of mass 10 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero

Moments about D: 20 g × (2.5 − 2) = 10 g × x

20 × 0.5 = 10 x

Answer: 1 m

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -5 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-5 + 2\lambda = 3 + (-2)\mu$

j: $0 + 1\lambda = 1 + 2\mu$

Solving i and j: $\lambda = 3$, $\mu = 1$

k check: $-1 + 0(3) = -1$ and $-1 + 0(1) = -1$, equal

Point: $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{1} \\ \mathbf{3} \\ \mathbf{-1} \end{pmatrix}$

10 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $x^2 - y^2 = 34$.

Assume $x^2 - y^2 = 34$, so $(x - y)(x + y) = 34$.

$x - y$ and $x + y$ have the same parity (their sum 2x is even).

If both are odd, the product is odd; if both are even, the product is a multiple of 4.

But 34 is even and is not a multiple of 4 (34 = 4 × 8 + 2), a contradiction.

So no such integers exist.

Answer: no integer solutions (product is odd or a multiple of 4)