Day 50 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Trig equations in radians (All boards)

Solve $\tan 2x = 2$ for $0 \leq x \leq \pi$,
giving answers to 3 significant figures.

Let $\theta = 2x$, so $\tan \theta = 2$ for $0 \leq \theta \leq 2\pi$
Calculator: $\theta = 1.107$, then use the symmetry of the graph
$\theta = 1.107, 4.249$
$x = \frac{\theta}{2}$
$x = 0.5536, 2.124$
Answer: x = 0.554, 2.12

2 Discrete random variable (AQA, OCR, MEI, Eduqas, CIE)

$P(X = x) = kx$ for $x = 1, 2, ..., 4$.
(a) Find k.
(b) Find E(X) and Var(X).

(a) $k(1 + 2 + 3 + 4) = 1$, so $10k = 1$ and $k = \frac{1}{10}$

(b) $E(X) = 1 \times \frac{1}{10} + 2 \times \frac{1}{5} + 3 \times \frac{3}{10} + 4 \times \frac{2}{5} = 3$

$E(X^2) = 10$, so $Var(X) = 10 - (3)^2 = 1$

Answer: (a) $k = \frac{1}{10}$ (b) E(X) = 3, Var(X) = 1

3 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 4 s, (b) the distance between the stations.

(a) a = gradient = 10 ÷ 4
(b) Distance = area of trapezium = ½ × (24 + 10) × 10

Answer: (a) 2.5 m s^{-2} (b) 170 m

4 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre $(-4, -2)$ and passes through the point $(-4, 2)$.
Find an equation of the circle.

$r^2 = (-4 - (-4))^2 + (2 - (-2))^2$
$= 0 + 16 = 16$

Answer: $(x + 4)^2 + (y + 2)^2 = 16$

5 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 4 kg is pulled across smooth horizontal ice by a rope with tension 25 N, inclined at α above the horizontal, where $\tan \alpha = \frac{4}{3}$.

Take g = 9.8 m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos \alpha = 3/5$, $\sin \alpha = 4/5$
(a) Horizontal: $25 \times 3/5 = 4a$
(b) Vertical: $R + 25 \times 4/5 = 4 \times 9.8$

Answer: (a) 3.75 m s^{-2} (b) 19.2 N

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6 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-2x-3}{(x+2)(x+3)}$ in partial fractions.

(b) Hence evaluate $\int_0^2 \frac{-2x-3}{(x+2)(x+3)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-2x-3}{(x+2)(x+3)} \equiv \frac{A}{x+2} + \frac{B}{x+3}$, so $-2x-3 \equiv A(x+3) + B(x+2)$
$x = -2$: $A = 1$; $x = -3$: $B = -3$

(b) $\int_0^2 \left(\frac{1}{x+2} + \frac{-3}{x+3}\right) dx = [\ln(x+2) - 3\ln(x+3)]_0^2$
$= \ln\frac{4}{2} - 3\ln\frac{5}{3} = \ln\left(\frac{54}{125}\right)$

Answer: (a) A = 1, B = −3 (b) $\ln\left(\frac{54}{125}\right)$

7 Binomial mean and variance (All boards)

X has a binomial distribution with mean 4 and variance 3.2.
(a) Find n and p.
(b) Find P(X = 4) to 3 significant figures.

(a) $np = 4$ and $np(1-p) = 3.2$
$1 - p = 3.2 \div 4 = 0.8$, so $p = 0.2$ and $n = 4 \div 0.2 = 20$
(b) $P(X = 4) = {}^{20}C_4 \times 0.2^4 \times 0.8^{16} = 0.218$
Answer: (a) n = 20, p = 0.2 (b) 0.218

8 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\tan^2 x + 5\sec x + 7 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $\sec^2 x + 5\sec x + 6 = 0$
$(\sec x + 2)(\sec x + 3) = 0$, so $\sec x = -2$ or -3
$\sec x = -2$, so $\cos x = -\frac{1}{2}$: $x = 120.0°, 240.0°$
$\sec x = -3$, so $\cos x = -\frac{1}{3}$: $x = 109.5°, 250.5°$
Answer: x = 109.5°, 120.0°, 240.0°, 250.5°

9 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 - 2x - 3 > 0$.

Factorise: $(x-3)(x+1) > 0$
Critical values: $x = -1$ and $x = 3$
Sketch the ∪-shaped parabola: $x < -1$ or $x > 3$
Answer: x < −1 or x > 3

10 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a cube number cannot be written in the form 7k + 3, where k is an integer.

Write the integer as $n = 7q + j$, where $j = 0, 1, ..., 6$.
n^3 leaves the same remainder as j^3 when divided by 7.
$j = 0, 1, 2, 3, 4, 5, 6$ give remainders 0, 1, 1, 6, 1, 6, 6.
The only possible remainders are 0, 1, 6, never 3.
Answer: possible remainders 0, 1, 6, never 3