Day 5 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplify using double angle formulae (All boards)

Simplify $\frac{1 - \cos 2x}{2 \sin x}$,

giving your answer as a single trigonometric term.

$1 - \cos 2x = 2 \sin^2 x$

so the expression is $\frac{2 \sin^2 x}{2 \sin x} = \sin x$

Answer: sin x

2 Interpreting regression (AQA, Edexcel, OCR, MEI, Eduqas)

Regression line (x from 2 to 10): $y = 3.2 + 0.8x$.
x is the advertising spend, y the weekly sales.
(a) Interpret the gradient.
(b) Estimate y when x = 10 and comment.

(a) The gradient: for each extra £100 of advertising, y is predicted to increase by 0.8
(b) $y = 3.2 + (0.8) \times 10 = 11.2$
x = 10 lies within the range of the data (interpolation), so the estimate is reliable
Answer: (a) y increases by 0.8 per extra £100 of advertising (b) 11.2, reliable (interpolation)

3 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = t^3 + 6t^2 - t$ metres.
Find its velocity and acceleration when t = 5.

$v = ds/dt = 3t^2 + 12t - 1$
$a = dv/dt = 6t + 12$
t = 5: v = 134, a = 42
Answer: $v = 134$ m s^{-1}, $a = 42$ m s^{-2}

4 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = \ln x + x - 5$. Use the Newton-Raphson method with
$x_0 = 4$ to find x_1 and x_2, to 4 d.p.

$f'(x) = 1 + \frac{1}{x}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 4 - \frac{0.3863}{1.250} = 3.6910$

$x_2 = 3.6910 - \frac{-0.003148}{1.271} = 3.6934$

Answer: $x_1 = 3.6910$, $x_2 = 3.6934$

5 Overtaking problem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A police car starts from rest and accelerates at 0.5 m s^{-2}. At that moment it is passed by a speeding van travelling at constant speed 17 m s^{-1}.
Find how long it takes to catch up, and the distance travelled.

Same distance: $\frac{1}{2} \times 0.5 \times t^2 = 17\ t$
$t(0.25\ t - 17) = 0$, $t \neq 0$ so $t = 17 \div 0.25$
Distance = 17 × 68
Answer: 68 s, 1156 m

6 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = x - 9$ meets the circle $x^2 + (y + 3)^2 = 50$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = x - 9$: $x^2 + (x - 6)^2 = 50$

$x^2 - 6x - 7 = 0$

$(x - 7)(x + 1) = 0$, so $x = -1$ or $x = 7$

$y = x - 9$: A = $(-1, -10)$, B = $(7, -2)$

$AB^2 = 8^2 + 8^2 = 128$

Answer: A$(-1, -10)$, B$(7, -2)$, AB = $8\sqrt{2}$

7 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = \frac{2y}{x}$ for $x > 0$,

given that $y = 3$ when $x = 1$.

$\frac{1}{y}\,dy = \frac{2}{x}\,dx$, so $\ln y = 2 \ln x + c$

$x = 1$, $y = 3$: $c = \ln 3$

$y = 3x^2$

Answer: $y = 3x^2$

8 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $x - 7y = 7$ and line l_2 has equation $x + 3y = -3$.

The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = $(0, -1)$

l_1 meets $y = 0$ at $x = 7$, and l_2 meets $y = 0$ at $x = -3$

Base = 10, height = 1

Area = $\frac{1}{2}$ × base × height = 5

Answer: 5

9 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ab^x$, where a and b are constants.

A plot of $\log_{10} y$ against x is a straight line with intercept 0.699 and gradient 0.699.

Find a and b to 2 significant figures.

$\log_{10} y = \log_{10} a + x \log_{10} b$

$\log_{10} a = 0.699$, so $a = 10^{0.699} = 5.0$

$\log_{10} b = 0.699$, so $b = 10^{0.699} = 5.0$

Answer: a = 5.0, b = 5.0

10 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(2 + x)^{-1}$ in ascending powers of x up to and including the term in x^3.

(b) State the range of values of x for which the expansion is valid.

Take out 2: $(2 + x)^{-1} = 2^{-1}(1 + \frac{1}{2}x)^{-1} = \frac{1}{2}(1 + \frac{1}{2}x)^{-1}$

Expand $(1 + \frac{1}{2}x)^{-1} = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$

Multiplying by $\frac{1}{2}$: coefficients of 1, x, x^2, x^3 are $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}$

Valid when $|\frac{1}{2}x| < 1$, that is $|x| < 2$

Answer: (a) $\frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots$ (b) $|x| < 2$