**Day 49** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Binomial expected frequency (All boards)

A fair die is rolled 11 times and the number of sixes is X.
This is done 500 times. Estimate the number of times that X = 4.

$X \sim B(11, \frac{1}{6})$

$P(X = 4) = {}^{11}C_4 \times (\frac{1}{6})^4 \times (\frac{5}{6})^7 = 0.07106$

Expected = 500 × 0.07106 = 35.53

**Answer: about 36 times**

## 2 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $3\sin^2 x - 2\cos x - 3 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\sin^2 x = 1 - \cos^2 x$: $-3\cos^2 x - 2\cos x = 0$, i.e. $3\cos^2 x + 2\cos x = 0$

$(\cos x)(3\cos x + 2) = 0$, so $\cos x = 0$ or $-\frac{2}{3}$

$\cos x = 0$: $x = 90.0°, 270.0°$

$\cos x = -\frac{2}{3}$: $x = 131.8°, 228.2°$

**Answer: x = 90.0°, 131.8°, 228.2°, 270.0°**

## 3 Power, driving force and resistance (CIE)

A car of mass 1000 kg moves along a horizontal road. The engine works at a constant rate of 36 kW. The resistance to motion is constant at 1000 N.
Find the acceleration when the speed is 25 m $s^{-1}$.

P = Fv: F = 36000 ÷ 25 = 1440 N
F − R = ma: 1440 − 1000 = 1000 a

**Answer: 0.44 m $s^{-2}$**

## 4 Fixed-point iteration (All boards)

(a) Show that $e^x - x - 5 = 0$ can be written as $x = \ln(x + 5)$.
(b) Use $x_{n+1} = \ln(x_n + 5)$ with $x_0 = 1$ to find $x_1$, $x_2$, $x_3$ to 4 d.p.

(a) $e^x = x + 5$, so $x = \ln(x + 5)$
(b) $x_1 = 1.7918$
$x_2 = 1.9157$
$x_3 = 1.9338$

**Answer: $x_1$ = 1.7918, $x_2$ = 1.9157, $x_3$ = 1.9338**

## 5 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 20, frequency 24, is a bar 5 cm wide, 4.8 cm tall.
(a) Find the bar height for width 10, frequency 12.
(b) Find the frequency of a 1.5 cm by 4 cm bar.

Area of bar ∝ frequency: 5 × 4.8 = 24 $cm^2$ represents 24
so 1 unit of frequency = 1 $cm^2$
(a) Width = 2.5 cm, area = 1 × 12 = 12 $cm^2$, height = 12 ÷ 2.5 = 4.8 cm
(b) Area = 1.5 × 4 = 6 $cm^2$, frequency = 6 ÷ 1 = 6

**Answer: (a) 4.8 cm (b) 6**

**Day 49** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 6 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{2x - 3}$ for $x \geq \frac{3}{2}$.

Find $f^{-1}(x)$ and state its domain.

Let $y = \sqrt{2x - 3}$. Then $y^2 = 2x - 3$

$x = \frac{y^2 + 3}{2}$

$f(x) \geq 0$, so the domain of $f^{-1}$ is $x \geq 0$.

**Answer: $f^{-1}(x) = \frac{x^2 + 3}{2}$, domain $x \geq 0$**

## 7 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 6 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles $\alpha$ and $\beta$ with the ceiling, where $\tan\alpha = \frac{4}{3}$ and $\tan\beta = \frac{3}{4}$.

Take $g = 9.8$ m s$^{-2}$.
Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{3}{5} = T_2 \times \frac{4}{5}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 58.8$

From the first: $T_1 = \frac{4}{3} T_2$; into the second: $T_2 \times \frac{5}{3} = 58.8$

**Answer: $T_1 = 47.04$ N (at $\alpha$); $T_2 = 35.28$ N (at $\beta$)**

## 8 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass M kg of a substance satisfies $\frac{dM}{dt} = -0.1M$, where t is in years.

Initially $M = 40$. Find M when $t = 10$, and the half-life, each to 3 s.f.

$\frac{1}{M} dM = -0.1\, dt$, so $\ln M = -0.1t + c$, giving $M = 40e^{-0.1t}$

$t = 10$: $M = 40\, e^{-1} = 14.72$

Half-life: $e^{-0.1t} = \frac{1}{2}$, so $t = \frac{\ln 2}{0.1} = 6.931$

**Answer: M = 14.7 kg, half-life 6.93 years**

## 9 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $2^{2x} - 3\times2^x + 2 = 0$.

Let $y = 2^x$. Then $2^{2x} = y^2$, so $y^2 - 3y + 2 = 0$
$(y - 1)(y - 2) = 0$
$y = 1$ or $y = 2$
$2^x = 1$ gives $x = 0$; $2^x = 2$ gives $x = 1$.
**Answer: x = 0 or x = 1**

## 10 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
$2x + y = 15$
$x^2 + y^2 = 65$

$y = -2x + 15$; substitute: $x^2 + (-2x + 15)^2 = 65$
$5x^2 - 60x + 160 = 0$, so $x^2 - 12x + 32 = 0$
$(x - 4)(x - 8) = 0$, so $x = 4$ or $x = 8$
$y = 15 - 2x$ gives $y = 7$ and $y = -1$
**Answer: x = 4, y = 7 and x = 8, y = −1**