Day 48 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(-3\mathbf{i})$ m s^{-1} at time t = 0 and moves with constant acceleration $(\mathbf{j})$ m s^{-2}. Find its velocity and speed when t = 4.

$v = u + at = (-3\mathbf{i}) + 4(\mathbf{j})$

$= -3\mathbf{i} + 4\mathbf{j}$

Speed $= \sqrt{(-3^2 + 4^2)}$

Answer: $\mathbf{v} = (-3\mathbf{i} + 4\mathbf{j})$ m s^{-1}; speed 5 m s^{-1}

2 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^1 xe^{2x}\,dx$.

Let $u = x$ and $\frac{dv}{dx} = e^{2x}$, so $\frac{du}{dx} = 1$ and $v = \frac{e^{2x}}{2}$

Integral $= [uv] - \int v\frac{du}{dx}\,dx$

$= \left[\frac{xe^{2x}}{2}\right]_0^1 - \int_0^1 \frac{e^{2x}}{2}\,dx$

$= \frac{1}{4} + \frac{e^2}{4}$

Answer: $\frac{1}{4} + \frac{e^2}{4}$

3 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{12 + 2\sqrt{3}}{\sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are integers.

Multiply top and bottom by $\sqrt{3}$:

$\frac{(12 + 2\sqrt{3})\sqrt{3}}{3} = \frac{12\sqrt{3} + 6}{3}$

$= 2 + 4\sqrt{3}$

Answer: $2 + 4\sqrt{3}$

4 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 4 m and mass 10 kg hangs horizontally from two vertical strings attached 1.5 m from A and 1.5 m from B.

A particle of mass 5 kg is attached 2 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(1) = 10\,g(0.5) + 5\,g(0.5)$

$T_2 = 73.5$

$T_1 + T_2 = 15\,g$ so $T_1 = 147 - 73.5$

Answer: Near A: 73.5 N; near B: 73.5 N

5 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 192; systematic sample of size 16.

(a) Find the sampling interval.

(b) The first item is number 12. Find the 3-th item.

(a) Interval $= 192 \div 16 = 12$

(b) 3-th item $= 12 + 2 \times 12 = 36$

Answer: interval 12; item number 36

6 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A new car costs £20000. Its value falls by 15% each year.
(a) Find its value after 3 years (nearest £).
(b) In which year does it first fall below £3000?

(a) Value = $20000 \times 0.85^3 = £12{,}283$

(b) $20000 \times 0.85^n < 3000$, so $n \lg 0.85 < \lg(3000 \div 20000)$
$n > \lg(3000 \div 20000) \div \lg 0.85 = 11.673$ (dividing by a negative number)
So $n = 12$

Answer: (a) £12,283 (b) 12 years

7 Inequalities with exponents and logs (All boards)

£1000 is invested at 10% compound interest per year.
Find the least whole number of years for the investment to exceed £2000.

$1000 \times 1.1^n > 2000$, so $1.1^n > 2$

$n > \frac{\ln 2}{\ln 1.1} = 7.273$

The least whole number is $n = 8$

Answer: 8 years

8 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.2. It is tossed 40 times and lands heads 4 times.
Test, at the 5% significance level, whether the probability of heads is different from 0.2.

H_0: $p = 0.2$, H_1: $p \neq 0.2$. Under H_0, $X \sim B(40, 0.2)$

$P(X \leq 4) = 0.0759$

Two-tailed: compare with 0.025. $0.0759 > 0.025$, so do not reject H_0

There is insufficient evidence that the probability of heads is different from 0.2

Answer: do not reject H_0: insufficient evidence that the probability of heads is different from 0.2

9 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 3x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$

Here $n = \frac{1}{3}$ and $a = 3$:

coefficients: x: 1, x^2: -1, x^3: $\frac{5}{3}$

Valid when $|3x| < 1$, that is $|x| < \frac{1}{3}$

Answer: (a) $1 + x - x^2 + \frac{5}{3}x^3 + \ldots$ (b) $|x| < \frac{1}{3}$

10 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find y in terms of x, given that $\frac{dy}{dx} = \frac{6x}{y}$
and $y = 1$ when $x = 0$.

$y\,dy = 6x\,dx$, so $\frac{y^2}{2} = \frac{6x^2}{2} + c$

$x = 0$, $y = 1$: $c = 0.5$

$y^2 = 6x^2 + 1$, so $y = \sqrt{6x^2 + 1}$ $(y > 0)$

Answer: $y = \sqrt{6x^2 + 1}$