Day 47 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix}-4\\2\end{pmatrix}$ and B has position vector $\begin{pmatrix}-10\\-4\end{pmatrix}$.

The point P lies on AB with AP : PB = 1 : 1.

Find the position vector of P.

$\text{AB} = \begin{pmatrix}-6\\-6\end{pmatrix}$

$\text{AP} = \frac{1}{2}\text{AB} = \begin{pmatrix}-3\\-3\end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix}-7\\-1\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{-7}\\\mathbf{-1}\end{pmatrix}$

2 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-4, -6)$ and $(8, 2)$.

(a) Find the midpoint of AB.

(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{-4+8}{2}, \frac{-6+2}{2}\right) = (2, -2)$

(b) Gradient $= \frac{8}{12} = \frac{2}{3}$

$y + 6 = \frac{2}{3}(x + 4)$

Multiply out and collect terms: $2x - 3y - 10 = 0$

Answer: (a) (2, −2) (b) $2x - 3y - 10 = 0$

3 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 3e^{x} - 10 = 0$,

giving any answers as exact values.

Let $y = e^{x}$: $y^2 - 3y - 10 = 0$

$(y + 2)(y - 5) = 0$

$e^{x} > 0$, so $y = -2$ is rejected; $e^{x} = 5$

$x = \ln 5$

Answer: $x = \ln 5$

4 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(2\mathbf{i} + 4\mathbf{j})$ m s^{-1} at time t = 0 and moves with constant acceleration $(\mathbf{i} + \mathbf{j})$ m s^{-2}.

Find its velocity and speed when t = 4.

$\mathbf{v} = \mathbf{u} + \mathbf{at} = (2\mathbf{i} + 4\mathbf{j}) + 4(\mathbf{i} + \mathbf{j})$

$= 6\mathbf{i} + 8\mathbf{j}$

Speed $= \sqrt{(6^2 + 8^2)}$

Answer: $\mathbf{v} = (6\mathbf{i} + 8\mathbf{j})$ m s^{-1}; speed 10 m s^{-1}

5 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n + \frac{15}{2}$, $u_1 = 18$.

(a) Find u_2.

(b) Given that the sequence converges to a limit L, find L.

(a) $u_2 = -\frac{1}{2} \times 18 + \frac{15}{2} = -\frac{3}{2}$

(b) As $n \to \infty$, u_{n+1} and u_n both tend to L

$L = -\frac{1}{2}L + \frac{15}{2}$

$L(1 - (-\frac{1}{2})) = \frac{15}{2}$, so $L = 5$

Answer: (a) $u_2 = -\frac{3}{2}$ (b) $L = 5$

6 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-x - 6}{(x + 3)(x + 4)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{-x - 6}{(x + 3)(x + 4)}\,dx$,

giving the answer as a single logarithm.

(a) $\frac{-x - 6}{(x + 3)(x + 4)} \equiv \frac{A}{x + 3} + \frac{B}{x + 4}$, so $-x - 6 \equiv A(x + 4) + B(x + 3)$

$x = -3$: $A = -3$; $x = -4$: $B = 2$

(b) $\int_0^4 (\frac{-3}{x + 3} + \frac{2}{x + 4})\,dx = [-3\ln(x + 3) + 2\ln(x + 4)]_0^4$

$= -3\ln\frac{7}{3} + 2\ln\frac{8}{4} = \ln(\frac{108}{343})$

Answer: (a) $A = -3$, $B = 2$ (b) $\ln(\frac{108}{343})$

7 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = 3t^3 + 5t^2 - t$ metres.
Find its velocity and acceleration when $t = 2$.

$v = ds/dt = 9t^2 + 10t - 1$

$a = dv/dt = 18t + 10$

$t = 2$: $v = 55$, $a = 46$

Answer: $v = 55$ m s^{-1}, $a = 46$ m s^{-2}

8 Binomial mean and variance (All boards)

X has a binomial distribution with mean 5 and variance 4.

(a) Find n and p.

(b) Find P(X = 5) to 3 significant figures.

(a) $np = 5$ and $np(1 - p) = 4$

$1 - p = 4 \div 5 = 0.8$, so $p = 0.2$ and $n = 5 \div 0.2 = 25$

(b) $P(X = 5) = {}^{25}C_5 \times 0.2^5 \times 0.8^{20} = 0.196$

Answer: (a) $n = 25$, $p = 0.2$ (b) 0.196

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Given that $\sqrt{3}$ is irrational, prove by contradiction that $2 + 2\sqrt{3}$ is irrational.

Assume $2 + 2\sqrt{3} = \frac{r}{s}$, where r and s are integers, $s \neq 0$.

Then $2\sqrt{3} = \frac{r}{s} - 2 = \frac{r - 2s}{s}$, so $\sqrt{3} = \frac{r - 2s}{2s}$.

The right-hand side is a ratio of integers, so it is rational.

This contradicts $\sqrt{3}$ being irrational, so $2 + 2\sqrt{3}$ is irrational.

Answer: $2 + 2\sqrt{3}$ is irrational

10 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.
P(X < 56) = 0.1587 and P(X > 68) = 0.0228.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.999$

$56 = \mu + (-0.9998)\sigma$ and $68 = \mu + 1.999\sigma$

Subtract: $12 = 2.999\,\sigma$, so $\sigma = 4.00$

$\mu = 56 - (-0.9998)(4.001) = 60.0$

Answer: $\mu = 60.0$, $\sigma = 4.00$