Day 46 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 10°C: $\frac{dT}{dt} = -k(T - 10)$. Initially T = 70 and after 10 minutes T = 50.

Find k to 3 s.f. and the time to cool to 20°C, to 3 s.f.

$\frac{1}{T-10}\,dT = -k\,dt$, so $\ln(T - 10) = -kt + c$, i.e. $T = 10 + 60e^{-kt}$

$t = 10$: $50 = 10 + 60e^{-10k}$, so $e^{-10k} = \frac{2}{3}$

$k = \frac{1}{10}\ln\frac{60}{40} = 0.04055$

$20 = 10 + 60e^{-kt}$: $t = \frac{1}{k}\ln\frac{60}{10} = 44.19$

Answer: k = 0.0405, t = 44.2 minutes

2 Smooth inclined plane (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A toy car slides from rest down a line of greatest slope of a smooth plane inclined at 30° to the horizontal.

Take $g = 9.8\ \text{m s}^{-2}$.

Find its acceleration, and the distance it travels in 3 s.

Along the plane: $mg\sin\alpha = ma$, so $a = 9.8 \times 0.5 = 4.9$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 4.9 \times 3^2$

Answer: 4.9 m s^{-2}; 22.05 m

3 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "6n + 11 is prime for every positive integer n."

Show that this statement is false by finding a counter-example.

Try n = 1, 2, 3, ... until the value is not prime.

n = 4 gives $6 \times 4 + 11 = 35$

$35 = 5 \times 7$, which is not prime.

Answer: n = 4 (value 35 is not prime)

4 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 3\,\text{cosec}\,x$ with respect to x,

and find the exact gradient when $x = \frac{\pi}{6}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = -3\cot x\,\text{cosec}\,x$

At $x = \frac{\pi}{6}$: $\frac{dy}{dx} = -6\sqrt{3}$

Answer: $\frac{dy}{dx} = -3\cot x\,\text{cosec}\,x$; gradient $-6\sqrt{3}$

5 Venn diagram with independent events (All boards)

The Venn diagram shows probabilities for events A and B, which are independent.

Find the values of x and y.

P(A) = 0.225 + 0.075 = 0.3 and $P(A \cap B) = 0.075$

Independent: $P(A \cap B) = P(A)P(B)$, so $P(B) = 0.075 \div 0.3 = 0.25$

$x = P(B) - P(A \cap B) = 0.25 - 0.075 = 0.175$

$y = 1 - (0.225 + 0.075 + 0.175) = 0.525$

Answer: x = 0.175, y = 0.525

6 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (2t^2 + 4)\mathbf{i} + (5t - 20)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0
$5t - 20 = 0$, so $t = 4$
$\mathbf{v} = (2 \times 4^2 + 4)\mathbf{i} = 36\mathbf{i}$
Answer: t = 4 s, speed 36 m s^{-1}

7 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = -\frac{3}{2}x - 10$. The point P is (0, 3).

(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l = $-\frac{3}{2}$, so the perpendicular has gradient $\frac{2}{3}$

$y - 3 = \frac{2}{3}x$, giving $2x - 3y + 9 = 0$

(b) Solve $y = -\frac{3}{2}x - 10$ with $2x - 3y + 9 = 0$: F = (−6, −1)

$PF^2 = 6^2 + 4^2 = 52$
Answer: (a) 2x − 3y + 9 = 0 (b) F(−6, −1), PF = $2\sqrt{13}$

8 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \\ r \end{pmatrix}$ respectively.
Given that A, B, C are collinear, find r.

$\mathbf{AB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{AC} = \begin{pmatrix} -3 \\ 0 \\ r - -2 \end{pmatrix}$
$\mathbf{AC} = \lambda\mathbf{AB}$: component 1 gives $-3 = \lambda \times 3$, so $\lambda = -1$
$r - -2 = -1 \times 1$, so $r = -3$
Answer: r = −3

9 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = x - 9$ meets the circle $(x - 2)^2 + (y - 2)^2 = 125$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = x - 9$: $(x - 2)^2 + (x - 11)^2 = 125$
$x^2 - 13x = 0$
$(x - 13)x = 0$, so $x = 0$ or $x = 13$
$y = x - 9$: A = (0, −9), B = (13, 4)
$AB^2 = 13^2 + 13^2 = 338$
Answer: A(0, −9), B(13, 4), AB = $13\sqrt{2}$

10 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
P(X < 75) = 0.1587 and P(X > 90) = 0.0228.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.999$
$75 = \mu + (-0.9998)\sigma$ and $90 = \mu + 1.999\sigma$
Subtract: $15 = 2.999\,\sigma$, so $\sigma = 5.00$
$\mu = 75 - (-0.9998)(5.002) = 80.0$
Answer: $\mu = 80.0$, $\sigma = 5.00$