Day 45 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Locating roots: change of sign (All boards)

The function $f(x) = x^3 - 1x - 2$.
Show that $f(x) = 0$ has a root between $x = 1.5$ and $x = 2$.

$f(1.5) = -0.125$
$f(2) = 4.00$
f is continuous and changes sign, so there is a root in the interval
Answer: $f(1.5) < 0$ and $f(2) > 0$: sign change

2 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 + 2x > 8$.

Rearrange: $x^2 + 2x - 8 > 0$
Factorise: $(x - 2)(x + 4) > 0$
Critical values: $x = -4$ and $x = 2$
Sketch the ∪-shaped parabola: $x < -4$ or $x > 2$
Answer: $x < -4$ or $x > 2$

3 Trig equations in radians (All boards)

Solve $\cos(x + 0.5) = -0.9$ for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

Let $\theta = x + 0.5$, so $\cos\theta = -0.9$ for $0.5 \leq \theta \leq 2\pi + 0.5$
Calculator: $\theta = 2.691$, then use the symmetry of the graph
$\theta = 2.691, 3.593$
$x = \theta - 0.5$
$x = 2.191, 3.093$
Answer: x = 2.19, 3.09

4 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the times taken (minutes) to complete a puzzle for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Boys: 35 − 15 = 20. Girls: 50 − 20 = 30
(b) Medians: 25 (Boys) and 35 (Girls), so Girls has the higher median
IQRs: Girls has the larger IQR, so its scores are more spread out
Answer: (a) 20 and 30 (b) Girls has the higher median; Girls is more spread out (larger IQR), Boys more consistent

5 Work-energy: speed after falling or sliding (CIE)

A particle of mass 3 kg slides from rest down a smooth curved slope, starting 10 m above the bottom.
Take $g = 9.8$ m s^{-2}.
Use energy to find its speed at the bottom of the slope.

Loss of PE = gain of KE: $m \times 9.8 \times 10 = \frac{1}{2} m v^2$
$v^2 = 19.6 \times 10 = 196$
$v = 14$
Answer: 14 m s^{-1}

6 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x + 3)^2 + (y - 1)^2 = 50$.
The point P(−8, 6) lies on C.
Find an equation of the tangent to C at P, in the form $ax + by + c = 0$ with integer a, b, c.

Centre of C = (−3, 1). Gradient of radius CP = $\frac{5}{-5} = -1$
The tangent is perpendicular to the radius: gradient = 1
$y - 6 = (x + 8)$
Multiply out and collect terms: $x - y + 14 = 0$
Answer: $x - y + 14 = 0$

7 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 6 \\ r \end{pmatrix}$ respectively.

Given that A, B, C are collinear, find r.

$\text{AB} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$, $\text{AC} = \begin{pmatrix} 0 \\ 4 \\ r - -2 \end{pmatrix}$

AC = λAB: component 2 gives $4 = \lambda \times 2$, so $\lambda = 2$

$r - -2 = 2 \times -3$, so $r = -8$

Answer: r = −8

8 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moves in a straight line with constant acceleration. Its final speed is 21 m s^{-1}, its acceleration is 1 m s^{-2} and the motion lasts 7 s.

Find the initial speed.

$v = u + at$

$21 = u + 1 \times 7$

$u = 21 - 7$

Answer: 14 m s^{-1}

9 Binomial two-tailed test (All boards)

A factory says a proportion 0.4 of its parts are defective. In a sample of 50 parts, 24 are defective.

Test, at the 5% significance level, whether the proportion is different from 0.4.

H_0: $p = 0.4$, H_1: $p \neq 0.4$. Under H_0, $X \sim B(50, 0.4)$

$P(X \geq 24) = 1 - P(X \leq 23) = 0.1562$

Two-tailed: compare with 0.025. 0.1562 > 0.025, so do not reject H_0

There is insufficient evidence that the proportion is different from 0.4

Answer: do not reject H_0: insufficient evidence that the proportion is different from 0.4

10 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of $(4 + 3x)^8$.

Term in $x^2 = 28 \times 4^6 \times (3x)^2$

$= 28 \times 4096 \times 9\, x^2$

Answer: 1032192