Day 44 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of 7 consecutive integers is always a multiple of 7.

Let the integers be n, n + 1, n + 2, n + 3, n + 4, n + 5, n + 6
Sum = 7n + 21
= 7(n + 3)
n is an integer, so (n + 3) is an integer, so the sum is a multiple of 7.
Answer: sum = 7(n + 3), a multiple of 7

2 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A boat moves in a straight line with constant acceleration. Its initial speed is 1 m s^{-1}, its final speed is 13 m s^{-1} and its acceleration is 4 m s^{-2}.
Find the time taken.

$v = u + at$
$13 = 1 + 4t$
$4t = 12$
Answer: 3 s

3 Conditional probability from a count (All boards)

40 customers: 12 buy coffee, 18 buy tea, 20 do neither.
(a) How many do both?
(b) Given a person who buys coffee, find P(this person buys tea).

(a) At least one: $40 - 20 = 20$. Both $= 12 + 18 - 20 = 10$

(b) Of the 12 who buy coffee, 10 also do the other: $P = \frac{10}{12} = \frac{5}{6}$

Answer: (a) 10 (b) $\frac{5}{6}$

4 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Expand and simplify $(-4 + 4\sqrt{5})(-5 + \sqrt{5})$, giving your answer in the form $a + b\sqrt{5}$.

$(-4 + 4\sqrt{5})(-5 + \sqrt{5}) = 20 - 4\sqrt{5} - 20\sqrt{5} + 4\sqrt{5}\times\sqrt{5}$
$\sqrt{5}\times\sqrt{5} = 5$, so the expression $= 40 - 24\sqrt{5}$
Answer: $40 - 24\sqrt{5}$

5 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The depth of water at a pier is modelled by $D = 4 + 4\sin(\frac{\pi t}{12})$, where t is in hours after noon.
Find (a) the greatest and least values, (b) the values of t with $0 \le t \le 24$ when it equals 6 m.

(a) sin ranges from -1 to 1: greatest $4 + 4 = 8$, least $4 - 4 = 0$

(b) $4 + 4\sin(\frac{\pi t}{12}) = 6$, so $\sin(\frac{\pi t}{12}) = \frac{1}{2}$

$\frac{\pi t}{12} = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, so $t = 2$ or 10
Answer: (a) 8 m, 0 m (b) t = 2 and 10

6 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x + 2)(x + 3)(x - 1) < 0$.

Critical values: $x = -3$, $x = -2$, $x = 1$
The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.
Reading from the sketch: $x < -3$ or $-2 < x < 1$
Answer: $x < -3$ or $-2 < x < 1$

7 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = 2t^3 + 4t^2$ metres.
Find (a) the average velocity over the first 4 seconds, (b) the velocity at t = 4.

(a) s(4) = 192, so average velocity = 192 ÷ 4

(b) $v = ds/dt = 6t^2 + 8t$, v(4) = 128

Answer: (a) 48 m s^{-1} (b) 128 m s^{-1}

8 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{2x}{64}$ for $0 \leq x \leq 8$, and 0 otherwise.

(a) Find P(X > 3).
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^2}{64}$

(a) $P(X > 3) = 1 - F(3) = \frac{55}{64}$

(b) $F(m) = 0.5$: $\frac{m^2}{64} = \frac{1}{2}$

m = 5.66

Answer: (a) $\frac{55}{64}$ (b) 5.66

9 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-4x - 19}{(x + 4)(x + 5)}$ in partial fractions.

(b) Hence evaluate $\int_0^3 \frac{-4x - 19}{(x + 4)(x + 5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-4x - 19}{(x + 4)(x + 5)} \equiv \frac{A}{x + 4} + \frac{B}{x + 5}$, so $-4x - 19 \equiv A(x + 5) + B(x + 4)$
x = −4: A = −3; x = −5: B = −1

(b) $\int_0^3 \left(\frac{-3}{x + 4} + \frac{-1}{x + 5}\right) dx = [-3\ln(x + 4) - \ln(x + 5)]_0^3$

$= -3\ln\frac{7}{4} - \ln\frac{8}{5} = \ln\left(\frac{40}{343}\right)$

Answer: (a) A = −3, B = −1 (b) $\ln\left(\frac{40}{343}\right)$

10 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 7 \\ -4 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $1 + 2\lambda = 7 + (-2)\mu$
j: $-1 + 3\lambda = -4 + 3\mu$
Solving i and j: $\lambda = 1$, $\mu = 2$
k check: $-2 + 3(1) = 1$ and $-3 + 2(2) = 1$, equal

Point: $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{3} \\ \mathbf{2} \\ \mathbf{1} \end{pmatrix}$