Day 43 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lorry of mass 800 kg moves along a straight horizontal road. The engine produces a driving force of 1500 N and the resistance to motion is 500 N.
Find the acceleration of the lorry.

Resultant force = 1500 − 500 = 1000 N
F = ma: 1000 = 800 a
Answer: 1.25 m s^{-2}

2 Basic trig equations (degrees) (All boards)

Solve $\sin x = -\frac{\sqrt{3}}{2}$ for $0 \leq x \leq 360°$.

Principal value $x = -60°$
Second solution: $180° - -60° = 240°$
Answer: x = 240°, 300°

3 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + kx^2 + 38x - 24$.
Given that $(x - 1)$ is a factor of $f(x)$,
(a) find the value of k,
(b) factorise $f(x)$ completely.

$f(1) = 0$: $2 + k + 38 - 24 = 0$
$k = -16$
$f(x) = (x - 1)(2x^2 - 14x + 24)$
$= 2(x - 4)(x - 3)(x - 1)$
Answer: (a) k = −16 (b) f(x) = 2(x − 4)(x − 3)(x − 1)

4 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 5 \tan x$ with respect to x,
and find the exact gradient when $x = \frac{\pi}{3}$.

Use the standard results for tan, sec, cosec and cot with the chain rule
$\frac{dy}{dx} = 5 \sec^2 x$
At $x = \frac{\pi}{3}$: $\frac{dy}{dx} = 20$
Answer: $\frac{dy}{dx} = 5 \sec^2 x$; gradient 20

5 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(2 + 2x)(1 + 3x)^7$.

$(1 + 3x)^7$ has x^1 coefficient 21 and x^2 coefficient 189
x^2 term of the product = 2 × 189 + 2 × 21
= 420
Answer: 420

6 Laws of logarithms: solving equations (All boards)

Solve $\log_{10} x + \log_{10}(x + 9) = 1$.

$\log_{10}(x(x + 9)) = 1$, so $x(x + 9) = 10^1 = 10$
$x^2 + 9x - 10 = 0$, so $(x - 1)(x + 10) = 0$
$x > 0$ for the logs to exist, so $x = 1$
Answer: x = 1

7 Finding the mean or standard deviation (All boards)

The mass X g of a flour bag is normal with mean 250.
Given P(X < 250.8) = 0.95, find the standard deviation (3 s.f.).

$z = 1.645$, so $\frac{250.8 - 250}{\sigma} = 1.645$

$\sigma = 0.8 \div 1.645 = 0.486$

Answer: $\sigma = 0.486$

8 Horizontal projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A package is thrown horizontally at 8 m s^{-1} from the top of a building 11.025 m above level ground.
Take g = 9.8 m s^{-2}.
Find the time of flight and the horizontal distance from the base where it lands.

Vertical: $11.025 = \frac{1}{2} \times 9.8 \times t^2$, $t^2 = 2.25$
$t = 1.5$ s
Horizontal: $x = 8 \times 1.5$
Answer: 1.5 s, 12 m

9 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.2. In a sample of 50 students, 3 pass first time.
Test, at the 5% significance level, whether the probability is different from 0.2.

H_0: $p = 0.2$, H_1: $p \neq 0.2$. Under H_0, $X \sim B(50, 0.2)$
$P(X \leq 3) = 0.0057$
Two-tailed: compare with 0.025. $0.0057 < 0.025$, so reject H_0
There is sufficient evidence that the probability is different from 0.2
Answer: reject H_0: sufficient evidence that the probability is different from 0.2

10 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{-4x - 4}{3x - 4}$ for $x \in \mathbb{R}$, $x \neq \frac{4}{3}$.
(a) Find $f^{-1}(x)$.
(b) State the domain of f^{-1}.

Let $y = \frac{-4x - 4}{3x - 4}$. Then $y(3x - 4) = -4x - 4$
$3xy - 4y = -4x - 4$, so $x(3y + 4) = 4y - 4$
$f^{-1}(x) = \frac{4x - 4}{3x + 4}$

f never takes the value $-\frac{4}{3}$, so the domain of f^{-1} is $x \neq -\frac{4}{3}$.

Answer: (a) $f^{-1}(x) = \frac{4x - 4}{3x + 4}$ (b) $x \neq -\frac{4}{3}$