Day 42 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
$2x + y = -10$
$x^2 + y^2 = 65$

$y = -2x - 10$; substitute: $x^2 + (-2x - 10)^2 = 65$
$5x^2 + 40x + 35 = 0$, so $x^2 + 8x + 7 = 0$
$(x + 1)(x + 7) = 0$, so $x = -1$ or $x = -7$
$y = -10 - 2x$ gives $y = -8$ and $y = 4$
Answer: $x = -7$, $y = 4$ and $x = -1$, $y = -8$

2 Normal distribution probabilities (All boards)

The mass X g of a flour bag is normal with mean 170, s.d. 8.
Find, to 3 s.f., the probability a value is
(a) less than 174,
(b) between 174 and 178.

$X \sim N(170, 8^2)$
(a) $P(X < 174) = 0.691$
(b) $P(174 < X < 178) = 0.8413 - 0.6915 = 0.150$
Answer: (a) 0.691 (b) 0.150

3 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $8^{\frac{1}{3}}$.

$8 = 2^3$, so $8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2^1$
$= 2$
Answer: 2

4 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = xe^{-2x}$,
and determine its nature.

$\frac{dy}{dx} = (1 - 2x)e^{-2x} = 0$
$x = \frac{1}{2}$
$\frac{d^2y}{dx^2} = 4(x - 1)e^{-2x}$
At $x = \frac{1}{2}$: $\frac{d^2y}{dx^2} = -\frac{2}{e} < 0$, so a maximum
Answer: $(\frac{1}{2}, \frac{1}{2e})$ maximum

5 Conditional probability formula (All boards)

P(B) = 0.3, P(A | B) = 0.2 and P(A) = 0.7.
(a) Find $P(A \cap B)$.
(b) Find P(B | A).
(c) Find $P(A \cup B)$.

(a) $P(A \cap B) = P(A \mid B) \times P(B) = 0.2 \times 0.3 = 0.06$
(b) $P(B \mid A) = \frac{0.06}{0.7} = \frac{3}{35}$
(c) $P(A \cup B) = 0.7 + 0.3 - 0.06 = 0.94$
Answer: (a) 0.06 (b) $\frac{3}{35}$ (c) 0.94

6 Equations using $\sin^2x + \cos^2x = 1$ (All boards)

Solve $7 \sin x + 4 \cos^2x - 7 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2x = 1 - \sin^2x$: $7 \sin x - 4 \sin^2x - 3 = 0$, i.e. $4 \sin^2x + 3 - 7 \sin x = 0$

$(\sin x - 1)(4 \sin x - 3) = 0$, so $\sin x = 1$ or $\frac{3}{4}$

$\sin x = 1$: $x = 90.0°$

$\sin x = \frac{3}{4}$: $x = 48.6°, 131.4°$

Answer: $x = 48.6°, 90.0°, 131.4°$

7 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 10 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan \alpha = \frac{12}{5}$ and $\tan \beta = \frac{4}{3}$.

Take $g = 9.8$ m s^{-2}.
Find the tension in each string.

Horizontal: $T_1 \cos \alpha = T_2 \cos \beta$, $T_1 \times \frac{5}{13} = T_2 \times \frac{3}{5}$

Vertical: $T_1 \sin \alpha + T_2 \sin \beta = 98$

From the first: $T_1 = \frac{39}{25} T_2$; into the second: $T_2 \times \frac{56}{25} = 98$

Answer: $T_1 = 68.25$ N (at α); $T_2 = 43.75$ N (at β)

8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Given that $\sqrt{7}$ is irrational, prove by contradiction that $5 + 2\sqrt{7}$ is irrational.

Assume $5 + 2\sqrt{7} = \frac{r}{s}$, where r and s are integers, $s \neq 0$.

Then $2\sqrt{7} = \frac{r}{s} - 5 = \frac{r - 5s}{s}$, so $\sqrt{7} = \frac{r - 5s}{2s}$.

The right-hand side is a ratio of integers, so it is rational.

This contradicts $\sqrt{7}$ being irrational, so $5 + 2\sqrt{7}$ is irrational.

Answer: $5 + 2\sqrt{7}$ is irrational

9 Impulse of a constant force: $v = u + at$ with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A puck of mass 4 kg is moving on a smooth horizontal surface at 1 m s^{-1} when a constant horizontal force of 12 N, in the direction of motion, acts for 6 s.
Find the speed of the puck at the end of this time and the distance it moves.

$F = ma$: $a = 12 \div 4 = 3$

$v = u + at = 1 + 3 \times 6$

$s = ut + \frac{1}{2}at^2$

Answer: 19 m s^{-1}; 60 m

10 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 2 \tan x$ with respect to x,

and find the exact gradient when $x = \frac{\pi}{3}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = 2 \sec^2x$

At $x = \frac{\pi}{3}$: $\frac{dy}{dx} = 8$

Answer: $\frac{dy}{dx} = 2 \sec^2x$; gradient 8