Day 41 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-1 \leq -3x + 4 < 10$.

Subtract 4 from each part: $-5 \leq -3x < 6$

Divide each part by -3 (reversing the inequalities): $-2 < x \leq \frac{5}{3}$

Answer: $-2 < x \leq \frac{5}{3}$

2 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 300 kg is raised and lowered by a vertical cable. The lift accelerates upwards at 0.5 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolve vertically (up positive): $T - 300 \times 9.8 = 300 \times 0.5$
$T = 300(9.8 + 0.5)$
Answer: 3090 N

3 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = 2$ and θ is acute,
find the exact value of $\sec\theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \text{cosec}^2\theta$
$\tan\theta = 2$, so $\sec^2\theta = 1 + 4 = 5$
$\sec\theta = \sqrt{5}$ (positive as θ is acute)
Answer: $\sqrt{5}$

4 Large data set: outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Large data set: mean windspeed (knots) at Leeming has mean 9, sd 4.
Outliers are more than 2 sd from the mean.
Are the values 16 and -2 outliers?

Limits: $9 \pm 2 \times 4$, so from 1 to 17
16 is inside the limits, so it is not an outlier
-2 is outside the limits, so it is an outlier
Answer: 16: not an outlier; -2: outlier

5 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-2x - 5| < -3x + 5$.

Critical values: $-2x - 5 = -3x + 5$ gives $x = 10$; $-2x - 5 = -(-3x + 5)$ gives $x = 0$
Sketch $y = |-2x - 5|$ and $y = -3x + 5$, and compare the graphs at $x = 10$ and $x = 0$.
$|-2x - 5| < -3x + 5$ when $x < 0$
Answer: $x < 0$

6 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A chord AB subtends an angle of 1.9 radians at the centre O of a circle of radius 4 cm.
Find the perimeter of the minor segment, to 3 s.f.

Arc = $r\theta = 4 \times 1.9 = 7.6$

Chord AB = $2r \sin(\frac{\theta}{2}) = 6.507$

Perimeter = 7.6 + 6.507 = 14.11 cm
Answer: 14.1 cm

7 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 6 m and mass 40 kg rests horizontally on supports at C and D, where AC = 2 m and DB = 2 m.

A girl of mass 25 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero

Moments about D: $40\text{ g} \times (4 - 3) = 25\text{ g} \times x$

$40 \times 1 = 25\,x$

Answer: 1.6 m

8 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve y = f(x), where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

x-intercepts at $x = -4$, 1 and 4, so $y = k(x + 4)(x - 1)(x - 4)$

The curve passes through (0, 32): $k \times 4 \times (-1) \times (-4) = 32$

So $k = 2$

Answer: $f(x) = 2(x + 4)(x - 1)(x - 4)$

9 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write 5 cos θ + 12 sin θ in the form R cos(θ − α), with R > 0 and 0 < α < 90°.

(b) Hence find its maximum value and the smallest θ ≥ 0 at which it occurs. Give α to 1 d.p.

$R = \sqrt{5^2 + 12^2} = 13$, $\tan\alpha = \frac{12}{5}$ so $\alpha = 67.4°$

Maximum is R when $\cos(\theta - \alpha) = 1$, i.e. $\theta = \alpha$

Answer: (a) $13\cos(\theta - 67.4°)$ (b) max 13 at $\theta = 67.4°$

10 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 20, frequency 20, is a bar 2 cm wide, 5 cm tall.

(a) Find the bar height for width 10, frequency 18.

(b) Find the frequency of a 1 cm by 2.5 cm bar.

Area of bar ∝ frequency: $2 \times 5 = 10\text{ cm}^2$ represents 20

so 1 unit of frequency = 0.5 cm^2

(a) Width = 1 cm, area $= 0.5 \times 18 = 9\text{ cm}^2$, height $= 9 \div 1 = 9$ cm

(b) Area $= 1 \times 2.5 = 2.5\text{ cm}^2$, frequency $= 2.5 \div 0.5 = 5$

Answer: (a) 9 cm (b) 5