Day 40 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Independent and mutually exclusive events (All boards)

A and B are independent events with P(A) = 0.1 and P(A ∪ B) = 0.64.
(a) Find P(B).
(b) Find P(A' ∩ B').

(a) P(A ∪ B) = P(A) + P(B) − P(A)P(B)
0.64 = 0.1 + P(B)(1 − 0.1), so P(B) = 0.54 ÷ 0.9 = 0.6
(b) P(A' ∩ B') = 1 − P(A ∪ B) = 0.36
Answer: (a) 0.6 (b) 0.36

2 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 1000 kg is raised and lowered by a vertical cable. The lift accelerates upwards at 1.5 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolve vertically (up positive): T − 1000 × 9.8 = 1000 × 1.5
T = 1000(9.8 + 1.5)
Answer: 11300 N

3 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.
(a) Describe the correlation.
(b) Why should y = 6 + 1.5x not be used for x = 15?

(a) The points lie close to a straight line rising: strong positive correlation
(b) The data cover x = 1 to 10; x = 15 is outside this range
so using the line would be extrapolation and may be unreliable
Answer: (a) strong positive correlation (b) extrapolation outside the range of the data

4 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate

$\int_0^2 \sqrt{1 + x^3}\, dx$,

giving your answer to 3 significant figures.

$h = \frac{2 - 0}{4} = 0.5$

x: 0, 0.5, 1, 1.5, 2
y: 1.000, 1.061, 1.414, 2.092, 3.000

Area $\approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)]$

$= \frac{0.5}{2} \times [1.000 + 3.000 + 2 \times 4.567] = 3.283$

Answer: 3.28

5 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2^{4x} = 4^{3x - 3}$.

Write both sides as powers of 2: $2^{4x} = 2^{2(3x - 3)}$
Equate the indices: 4x = 6x − 6
−2x = −6
x = 3
Answer: x = 3

6 Trig equations in radians (All boards)

Solve sin x = 0.3 for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

sin θ = 0.3 for $0 \leq x \leq 2\pi$
Calculator: x = 0.3047, then use the symmetry of the graph
x = 0.3047, 2.837
Answer: x = 0.305, 2.84

7 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-6x - 22}{(x + 1)(x + 5)}$ in partial fractions.

$\frac{-6x - 22}{(x + 1)(x + 5)} \equiv \frac{A}{(x + 1)} + \frac{B}{(x + 5)}$, so $-6x - 22 \equiv A(x + 5) + B(x + 1)$

$x = -1$: $-16 = A \times 4$, so $A = -4$

$x = -5$: $8 = B \times (-4)$, so $B = -2$

Answer: $\mathbf{-\frac{4}{(x + 1)} - \frac{2}{(x + 5)}}$

8 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (2t^2 + 4)\mathbf{i} + (4t - 12)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0

$4t - 12 = 0$, so $t = 3$

$\mathbf{v} = (2 \times 3^2 + 4)\mathbf{i} = 22\mathbf{i}$

Answer: t = 3 s, speed 22 m s^{-1}

9 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 - 8x + 20$ for $x \geq 2$.
Find the range of f.

$f(x) = (x - 4)^2 + 4$, with minimum at $x = 4$

$x = 4$ is in the domain, so the minimum value is 4.

Answer: f(x) ≥ 4

10 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of sec 135°,
giving your answer with a rational denominator.

$\sec 135° = \frac{1}{\cos 135°}$

$\cos 135° = -\frac{\sqrt{2}}{2}$, so $\sec 135° = \frac{1}{-\frac{\sqrt{2}}{2}}$

Rationalising the denominator gives $-\sqrt{2}$

Answer: $\mathbf{-\sqrt{2}}$