Day 4 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-2x-4}{(x+1)(x+3)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{-2x-4}{(x+1)(x+3)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-2x-4}{(x+1)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+3}$, so $-2x-4 \equiv A(x+3) + B(x+1)$
$x = -1$: $A = -1$; $x = -3$: $B = -1$

(b) $\int_0^4 \left(\frac{-1}{x+1} + \frac{-1}{x+3}\right) dx = [-\ln(x+1) - \ln(x+3)]_0^4$
$= -\ln\frac{5}{1} - \ln\frac{7}{3} = \ln\left(\frac{3}{35}\right)$

Answer: (a) $A = -1$, $B = -1$ (b) $\ln\left(\frac{3}{35}\right)$

2 Work-energy: speed after falling or sliding (CIE)

A particle of mass 0.5 kg slides from rest down a smooth curved slope, starting 40 m above the bottom.
Take $g = 9.8$ m s^{-2}.
Use energy to find its speed at the bottom of the slope.

Loss of PE = gain of KE: $m \times 9.8 \times 40 = \frac{1}{2} m v^2$
$v^2 = 19.6 \times 40 = 784$
$v = 28$

Answer: 28 m s^{-1}

3 Union, conditional probability and independence (All boards)

P(A) = 0.25, P(B) = 0.35, P(A ∩ B) = 0.25.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) $P(A \cup B) = 0.25 + 0.35 - 0.25 = 0.35$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.35} = \frac{5}{7}$

(c) $P(A)P(B) = 0.25 \times 0.35 = 0.0875$, which does not equal $P(A \cap B)$

Answer: (a) 0.35 (b) $\frac{5}{7}$ (c) not independent

4 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A marble is projected vertically upwards from ground level with speed 29.4 m s^{-1}.
Take $g = 9.8$ m s^{-2}.
Find (a) the greatest height reached, (b) the total time before it returns to the ground.

(a) At the top $v = 0$: $0 = 29.4^2 - 2 \times 9.8 \times H$
$H = 864.36 \div 19.6 = 44.1$

(b) $s = 0$: $0 = 29.4\,t - 4.9t^2$, so $t = 29.4 \div 4.9$

Answer: (a) 44.1 m (b) 6 s

5 Completing the square proof (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $-2x^2 + 4x - 11 < 0$ for all real values of x.

Complete the square: $-2x^2 + 4x - 11 = -2(x-1)^2 - 9$
$(x-1)^2 \geq 0$ for all real x, so $-2(x-1)^2 \leq 0$.
Adding -9: the expression is ≤ -9, so < 0 for all real x.

Answer: $-2x^2 + 4x - 11 < 0$ for all x

6 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve y = sin x, the x-axis
and the lines x = 0 and x = π.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_0^{\pi} y^2 \, dx = \pi \int_0^{\pi} \sin^2 x \, dx$

$= \pi[\frac{x}{2} - \frac{\sin x \cos x}{2}]_0^{\pi}$

$= \frac{\pi^2}{2}$

Answer: $\mathbf{\frac{\pi^2}{2}}$

7 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The time, X minutes, to complete a task is normally distributed with mean 40 and s.d. 10.
A random sample of 16 is taken. Find the probability that the mean time is more than 45 (3 s.f.).

$\overline{X}$ ~ N(40, $\frac{10^2}{16}$), standard deviation 2.500

$z = \frac{45 - 40}{2.500} = 2.000$

P = 0.0228

Answer: 0.0228

8 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -10 \\ -12 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} -1 \\ -9 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-10 + 3\lambda = -1 + 0\mu$
j: $-12 + 3\lambda = -9 + 2\mu$
Solving i and j: $\lambda = 3$, $\mu = 3$
k check: $10 + (-2)(3) = 4$ and $7 + (-1)(3) = 4$, equal

Point: $\begin{pmatrix} -1 \\ -3 \\ 4 \end{pmatrix}$

Answer: $\mathbf{\begin{pmatrix} -1 \\ -3 \\ 4 \end{pmatrix}}$

9 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{15 + 2\sqrt{5}}{\sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers.

Multiply top and bottom by $\sqrt{5}$:

$\frac{(15 + 2\sqrt{5})\sqrt{5}}{5} = \frac{15\sqrt{5} + 10}{5}$

$= 2 + 3\sqrt{5}$

Answer: $\mathbf{2 + 3\sqrt{5}}$

10 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence has first term 100 and common ratio 0.5.
Find the smallest n for which the nth term is less than 1.

$100 \times 0.5^{n-1} < 1$
$(n - 1) \lg 0.5 < \lg(1 \div 100)$
lg 0.5 is negative, so dividing reverses the inequality:
$n - 1 > \lg(1 \div 100) \div \lg 0.5 = 6.644$, so $n > 7.644$
The smallest integer n is 8
Answer: n = 8