Day 39 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Binomial probabilities (All boards)

X ~ B(27, 0.7). Find, to 3 significant figures,
(a) P(X ≥ 19)
(b) P(X = 18)

(a) P(X ≥ 19) = 1 − P(X ≤ 18)
(a) = 0.577
(b) P(X = 18) = $^{27}C_{18} \times 0.7^{18} \times 0.3^{9} = 0.150$
Answer: (a) 0.577 (b) 0.150

2 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A radio-controlled car moves with constant velocity $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(4\mathbf{i} - 8\mathbf{j})$ m.
Find (a) its position vector when t = 6, (b) the value of t when it crosses the line through O in the direction of $\mathbf{i}$.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (4\mathbf{i} - 8\mathbf{j}) + 6(3\mathbf{i} + 4\mathbf{j}) = (22\mathbf{i} + 16\mathbf{j})$
(b) $\mathbf{j}$ component zero: −8 + 4t = 0, so t = 2
Answer: (a) (22i + 16j) m (b) t = 2 s

3 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 4x + 6 and g(x) = x^2 + 4 for x ∈ ℝ.
(a) Find fg(x).
(b) Solve gf(x) = 200.

fg(x) = f(x^2 + 4) = 4(x^2 + 4) + 6 = 4x^2 + 22
gf(x) = (4x + 6)^2 + 4 = 200
(4x + 6)^2 = 196, so 4x + 6 = ±14
x = −5 or x = 2
Answer: (a) fg(x) = 4x^2 + 22 (b) x = −5 or x = 2

4 Rates of change (connected rates) (All boards)

The area of a square increases at a constant rate of 6 cm^{2}s^{-1}.
Find the rate of increase of the side length when the side is 2 cm.

$A = x^2$, so $\frac{dA}{dt} = 2x\frac{dx}{dt}$
$6 = 2 \times 2 \times \frac{dx}{dt}$
$\frac{dx}{dt} = \frac{3}{2}$
Answer: $\frac{3}{2}$ cm/s

5 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point (1, 0) and B is the point (2, −5).
Find the perpendicular bisector of AB as ax + by + c = 0, with integer a, b, c.

Midpoint of AB = $(\frac{3}{2}, -\frac{5}{2})$
Gradient of AB = $\frac{-5}{1} = -5$
Perpendicular gradient = $\frac{1}{5}$
$y + \frac{5}{2} = \frac{1}{5}(x - \frac{3}{2})$, which gives x − 5y − 14 = 0
Answer: x − 5y − 14 = 0

6 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix}5\\4\\5\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}-4\\-2\\3\end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a}\cdot\mathbf{b} = (5)(-4) + (4)(-2) + (5)(3) = -13$

$|\mathbf{a}| = \sqrt{66}$, $|\mathbf{b}| = \sqrt{29}$

$\cos\theta = \frac{-13}{\sqrt{66}\sqrt{29}} = -0.2971$

$\theta = 107.3°$

Answer: 107.3°

7 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Given that $\sqrt{2}$ is irrational, prove by contradiction that $1 + 3\sqrt{2}$ is irrational.

Assume $1 + 3\sqrt{2} = \frac{r}{s}$, where r and s are integers, $s \neq 0$.

Then $3\sqrt{2} = \frac{r}{s} - 1 = \frac{r - 1s}{s}$, so $\sqrt{2} = \frac{r - 1s}{3s}$.

The right-hand side is a ratio of integers, so it is rational.

This contradicts $\sqrt{2}$ being irrational, so $1 + 3\sqrt{2}$ is irrational.

Answer: $1 + 3\sqrt{2}$ is irrational

8 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(60, 0.2). Use a normal approximation to find P(X ≥ 10), to 3 significant figures.

Y ~ N(np, np(1 − p)) = N(12.00, 9.600), so $\sigma = \sqrt{9.600} = 3.098$

Use a continuity correction.

P(X ≥ 10) ≈ P(Y > 9.5) = P(Z > −0.8069) = 0.790

Answer: 0.790

9 Quotient rule (All boards)

Find the x-coordinate(s) of the stationary point(s) of the curve $y = \frac{\ln x}{x}$.

$\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$

Stationary points where $\frac{dy}{dx} = 0$, i.e. the numerator is zero

Numerator: $1 - \ln x = 0$

$x = e$

Answer: $x = e$

10 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 6 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

The coefficient of friction is 0.25. Take g = 9.8 m s^{-2}. Find the acceleration down the plane, and the distance moved in 3 s.

$\cos\alpha = 4/5$, $\sin\alpha = 3/5$

$R = mg\cos\alpha$, $F = \mu R$

$ma = mg\sin\alpha - \mu mg\cos\alpha$, so $a = 9.8(3/5 - 0.25 \times 4/5)$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 3.92 \times 3^2$

Answer: 3.92 m s^{-2}; 17.64 m